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IS-LM Stability Revisited: Samuelson was Right, Modigliani was Wrong

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RESUMEN

En el modelo IS-LM de Hicks, en el que se asume que la producción se determina en el mercado de bienes y la tasa de interés en el mercado de dinero, cuando la propensión marginal a gastar es mayor que uno, la IS tiene pendiente positiva. Modigliani (1944), Varian (1977) y Sargent (1987), determinaron que en este caso especial el modelo IS-LM es estable cuando la pendiente de la LM es mayor que la de la IS.

En línea con Samuelson (1941), este artículo muestra que en este caso especial el modelo es estable cuando la pendiente de la IS es mayor que la de la LM. Sin embargo, en este caso estable, el modelo no tiene un significado económico útil.

Una solución a este problema teórico es abandonar el mecanismo de ajuste keynesiano y reemplazarlo por el mecanismo clásico, donde la tasa de interés se determina en el mercado de bienes y la producción en el mercado de dinero. En este caso el modelo IS-LM es estable cuando la LM es más empinada que la IS.

Palabras clave: Historia del pensamiento económico, teoría macroeconómica, modelo IS-LM.

Clasificación JEL: E11, B22.

La estabilidad de la IS-LM revisitada: Samuelson estaba en lo cierto, Modligiani equivocado

ABSTRACT

In Hicks's IS-LM model, where it is assumed that production is determined in the goods market and the interest rate is determined in the money market, when the marginal propensity to spend is greater than one, the IS has a positive slope. Modigliani (1944), Varian (1977) and Sargent (1987) determined that in this special case the IS-LM model is stable when the LM slope is greater than the IS.

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In line with Samuelson (1941), this article shows that in this case the model is stable when the IS slope is greater than the LM slope. However, in this stable case the model does not have a useful economic meaning.

One solution to this theoretical problem is to abandon the Keynesian adjustment mechanism and replace it with the Classical mechanism where the interest rate is determined in the goods market and production is determined in the money market. In this case, the IS-LM model is stable when the LM is steeper than the IS.

Keywords: Economic thinking, macroeconomic theory, IS-LM model.

JEL classification: E11, B22.

1. INTRODUCTION

In the traditional IS-LM model devised by Hicks (1937), when the marginal propensity to spend is greater than one, the Keynesian multiplier is negative and the IS has a positive slope. Hicks (1937) warned that in this special case, only under certain conditions can the IS-LM model be stable, and Keynes, in a letter published in Gilboy (1939), deemed this occurrence "completely unstable".

Later developments – starting with Modigliani (1944) and including articles in specialized journals, as well as sections on macroeconomics and mathematics in economics textbooks – agree that, in this atypical case, the IS-LM model is stable when the LM slopes more steeply than IS.

This article shows that:

i) The case the literature pronounces as stable is in fact unstable, because the same stability conditions that apply to the standard case are extended to the special case. The model is stable when the IS steeper than the LM. This result is consistent with the views expressed by Samuelson (1941).

ii) However, in this special stable case, the model's linear version provides results without useful economic significance. Firstly, the equilibrium values of production and the interest rate are negative. Secondly, since the Keynesian multiplier is negative, expanding demand results in shrinking output.

iii) One solution to this theoretical problem is to abandon the Keynesian adjustment mechanism and replace it with the Classical mechanism where the interest rate is determined in the goods market and production is determined in the money market. In this case the model is stable when the LM is steeper than the IS.

The following section reviews the literature on the special case. Section 3 discusses the stability conditions of the standard IS-LM model. Section 4 repeats the exercise for the special case and argues that the traditional treatment is incorrect. Section 5 proposes a complementary argument to show the instability of the case the literature deems stable, by simulating the dynamic effects of an expansive monetary policy. Section 6 shows that in the linear version of the stable model, the equilibrium values of the interest rate and output are negative, and that the model forecasts a reduction in output as a consequence of expanding demand. Section 7 shows that only in the case in which the Keynesian adjustment mechanism is replaced by the Classical mechanism is the stability condition postulated by Modigliani true. Finally, the conclusions are set out in Section 8.

2. BACKGROUND

In the IS-LM model, when the marginal propensity to spend (propensity to consume plus propensity to invest) is greater than one, the Keynesian multiplier is negative and the IS curve has a positive slope.

In Hicks's presentation (1937), equilibrium in a goods market requires ex ante equality between saving (S) and investment (I)[1]. Thus,

$$I(\overset{-}{i},\overset{+}{Y}) = S(\overset{+}{i},\overset{+}{Y})$$

The slope of the IS curve, graphed on the (Y, i) space, is given by:

$$\left.\frac{di}{dY}\right|_{IS} = -\frac{S_Y - I_Y}{S_i - I_i} = -\frac{1 - C_Y - I_Y}{S_i - I_i}$$

Where $Y_x = \partial Y / \partial x$ is the generic form representing the partial derivative of Y with respect to X.

The denominator is undoubtedly positive, while the numerator can be positive, in the typical case where the propensity to spend is less than one ($C_Y + I_Y < 1$), or negative, when the marginal propensity to spend is greater than one ($C_Y + I_Y > 1$).

One of the innovations in Keynes's *General Theory* was the introduction of the concepts of *marginal propensity to consume* and the *multiplier*. In a letter published in Gilboy (1939), Keynes warned that if the marginal propensity to consume is greater than one, the model would be unstable:

> My theory itself does not require my so-called psychological law as a premise. What the theory shows is that if the psychological law is *not* fulfilled, then we have a condition of complete instability. If, when incomes increase, expenditure increases by more than the whole of the increase in income, there is no point of equilibrium. (Gilboy, 1939, p. 634).

All the subsequent literature dealing explicitly with this special case (negative Keynesian multiplier and positively sloping IS) – that first appeared in specialized journals and was then adopted by macroeconomics and mathematics for inclusion in

[1] Hicks's text has been adapted to current terminology.

economists textbooks – has established that for the model to be stable, the LM curve must be steeper than the IS curve.

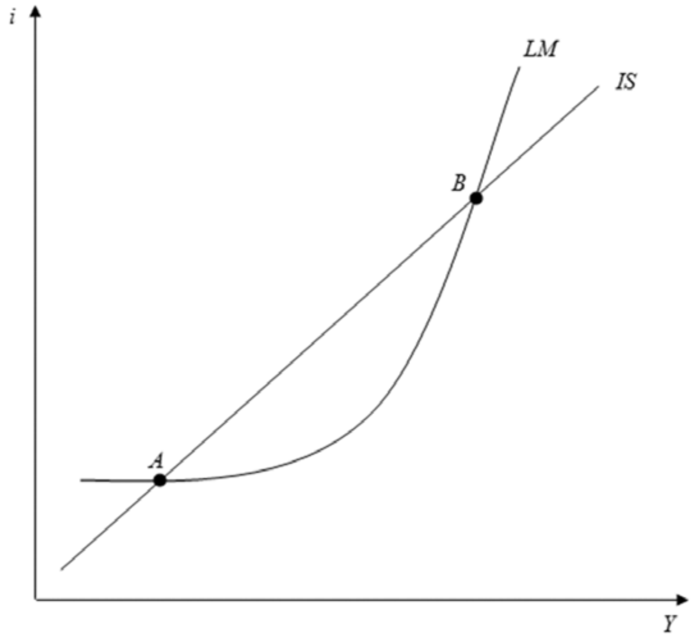
Modigliani (1944) was the first to address this special case in terms of the IS and LM slopes. For the case in which IS slopes positively, he states that:

> Stability is also possible when the IS curve rises in the neighborhood of the equilibrium points as long as it cuts the LL curve from its concave toward its convex side (Modigliani 1944, p. 64).

That is, for the model to be stable in the neighborhood of equilibrium, the LM slope must be greater than that of the IS.

This argument appears in Figure 1. The LM and IS intercept each other at points *A* and *B*. For Modigliani, the equilibrium point would be stable at *B*, where the IS cuts the LM from its concave to its convex side. At this point, the LM slope is greater than that of the IS. At point *A*, equilibrium would be unstable.

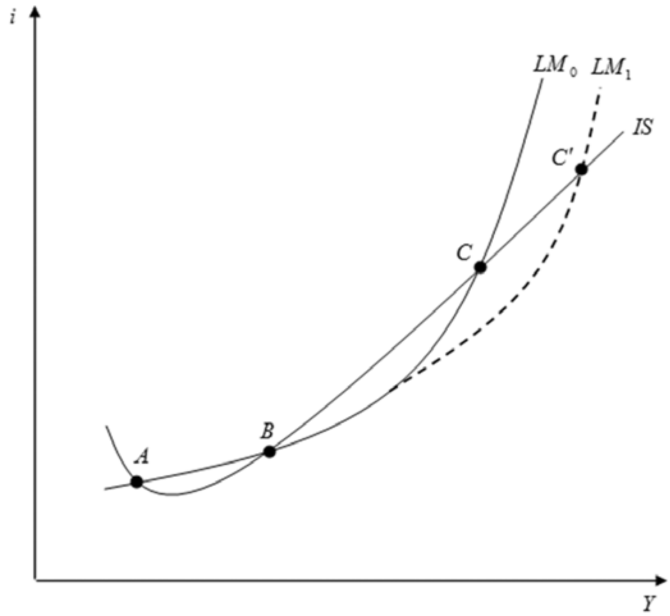
Figure 1



Later, Hudson (1957) presented the IS-LM model's stability condition for the special case and also erroneously considered the unstable case to be stable. Since Hudson's IS is non-linear and since it shows a negatively sloping stretch and another with a positive slope, Hudson's IS-LM model is one of multiple equilibrium, like the one shown in Figure 2. In this model:

Stability of equilibrium then requires that the IS schedule slopes upwards less steeply than the LM schedule. Consequently, *B* is a position of unstable equilibrium, while *C* and *C′* are stable. (Hudson, 1957, p. 382).

Figure 2



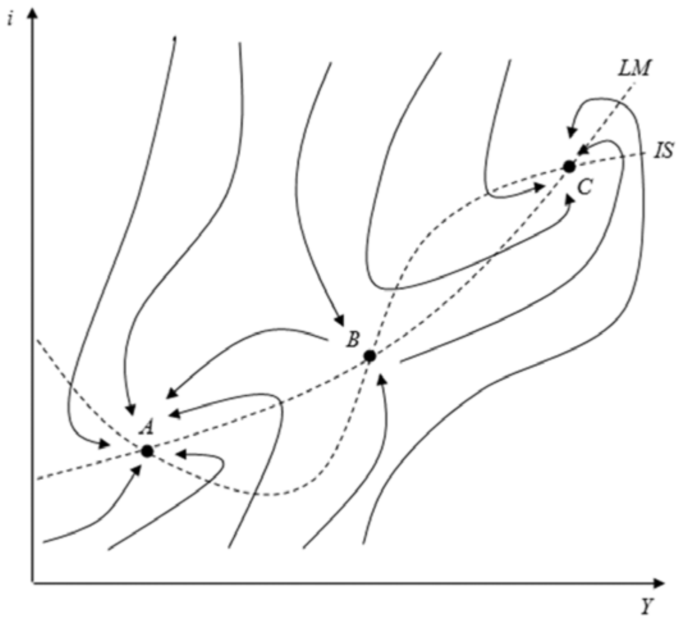
According to Hudson then, the atypical model is stable when the LM slope is greater than that of the IS.

Later, Dernburg and Dernburg (1969), Smith (1970), and Varian (1977) reached the same independent conclusion without referring to Modigliani's (1944) or to Hudson's work (1957).

Varian (1977) builds, as Hudson, a model with multiple equilibrium and arrives at the same result: when the IS slopes positively and there are multiple equilibria, the stable region is that where the LM is steeper than the IS.

Figure 3 – which reproduces Varian's Figure 3 (1977, p. 268) – shows, as Hudson, that the region is unstable (i.e. configures a *saddle point*) at the intersection points between the IS and LM, when the IS is steeper than the LM, as in point *B* of Figure 3. The intersection points are stable if the LM slope is greater than that of the IS (when the IS slopes negatively, as in point *A*, or when the IS slope is positive, but less than the LM, at point *C*), as shown by the direction arrows of the phase diagram.

Figure 3



Subsequent publications in specialized journals dealing with this special case leave these authors' proclaimed stability conditions unchanged, as shown in works by Silver (1971), Chang and Smyth (1972), Puckett (1973), Burrows (1974), Cebula (1980), Wang (1980), Patinkin (1987) and Ros (2004).

The special case and its stability conditions are also incorporated into the teaching of macroeconomics in both old (Smith, 1970, p. 313; Dernburg and McDougall, 1976, pp. 244-247; Branson, 1972, pp. 222-223), and more recent textbooks (Sargent, 1987, p. 59; McCafferty, 1990, p. 149).

When referring to the special case, Sargent states that:

> This condition is automatically satisfied when the LM curve is upward sloping and the IS curve is downward sloping. It can still be satisfied if the IS curve is upward sloping, provided that the LM curve is more steeply sloped. (Sargent, 1987, p. 59).

The same argument appears in text books on mathematics for economics, including Ferguson and Lim (1998, p. 129), Shone (2002, p. 449), and Zhang (2005, p. 243).

In short, all the literature – starting with Modigliani (1944) – states that in the case in which the IS has a positive slope, for the IS-LM to be stable, the LM curve must be steeper than the IS curve.

The following sections argue that the IS-LM model is stable in the special case where the IS slope is greater than the LM slope.

3. IS-LM STABILITY: THE STANDARD CASE

This section presents Hicks's model (1937), in its dynamic version[2]. In the goods market, it is assumed that adjustment is by quantities and that output increases ($\overset{0}{Y} > 0$) when there is an excess of demand in that market (*EDB*). In the money market, the adjustment variable is the interest rate, which rises ($i > 0$) when there is excess demand (*EDM*).

$$\overset{0}{Y} = \varepsilon(EDB); \varepsilon > 0 \tag{1}$$

$$\overset{0}{i} = \eta(EDM); \eta > 0 \tag{2}$$

Where ε and η are, respectively, the production adjustment velocities and the interest rate vis-à-vis excess demand in the goods market and the money market.

If Y represents output and Y^d stands for the demand for goods, in a closed economy without government, the demand for goods comes from consumption and private investment. Consumption and investment are positive functions of income and negative functions of the interest rate. Consequently, demand for goods is given by:

$$Y^d = C(\overset{-}{i}, \overset{+}{Y}) + I(\overset{-}{i}, \overset{+}{Y}); \ 0 < C_Y < 1 \tag{3}$$

Excess demand in the goods market therefore equals:

$$EDB = Y^d - Y = EDB(\overline{Y}, \overline{i}); \ EDB_Y = C_Y + I_Y - 1 < 0; \ EDB_i = C_i + I_i < 0 \tag{4}$$

In the money market, if M^s is the money supply and P is the price level, the real money supply equals:

$$m^s = \frac{M^s}{P} \tag{5}$$

If m^d is the real demand for money, which is directly related to output and inversely related to the interest rate:

$$m^d = m^d(\overset{+}{Y}, \overset{-}{i}) \tag{6}$$

Consequently, the excess demand in a money market equals:

$$EDM = m^d - m^s = EDM(\overset{+}{Y}, \overset{-}{i}); \ EDM_Y = m_Y^d > 0; EDM_i = m_i^d < 0. \tag{7}$$

[2] See Gandolfo (1996), Ferguson and Lim (1998) and Zhang (2005).

Replacing equations (7) and (4) in equations (1) and (2), respectively, we obtain a differential equation system to discuss the stability conditions of the traditional IS-LM model.

$$\overset{0}{Y} = \varepsilon\left[EDB(\overset{-}{Y},\overset{-}{i})\right];\varepsilon > 0 \tag{8}$$

$$\overset{0}{i} = \eta\left[EDM(\overset{+}{Y},\overset{-}{i})\right];\eta > 0 \tag{9}$$

Where $\varepsilon[.]$ and $\eta[.]$ are rising monotone functions which are differentiable and satisfy the $\varepsilon[0] = \eta[0] = 0$ condition.

To evaluate whether a model is stable or not, it is worth taking into account Gandolfo's warning (1996) about the appropriate formulation of adjustment dynamics in markets:

> The dynamic formalization of the Walrasian assumption is the following
>
> $$\overset{0}{p} = f\left[D(p) - S(p)\right]$$
>
> Where $\operatorname{sgn} f[...] = \operatorname{sgn}[...], f[0] = 0, f'(0) > 0$
>
> The notation $\operatorname{sgn} f[...] = \operatorname{sgn}[...]$ means that f is a *sign-preserving function*, i.e., the dependent variable has the same sign as the independent variable (which in this case is excess demand): therefore, if excess demand is positive (negative) the time derivative of p is positive (negative), i.e. p is increasing (decreasing). (Gandolfo, 1996, p.172).

Which, when applied to the IS, implies:

"$\overset{0}{Y} = \varphi_1\left[I(Y,i) - S(Y,i)\right]$, $\operatorname{sgn}\varphi_1[...] = \operatorname{sgn}[...]$" (Gandolfo, 1996, p. 328)[3].

Taking the case shown by Gandolfo as an example, since $\varphi_1 > 0$, the influence of the investment-saving $(I - S)$ gap on production, which equals the effect of the excess demand, also *must* be positive. In our presentation, since $\varepsilon > 0$, the influence of the high excess demand on the product *must* be positive.

The system made up by equations (8) and (9) is non-linear and its study implies analytical difficulties, which we will avoid. To this end, we will discuss some valid properties in a local equilibrium context, through Taylor's expansion.

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3 We have adapted Gandolfo's terminology to ours.

If Y^e and i^e represent stationary (local) equilibrium values of production and the interest rate in the IS-LM model, equations (8) and (9) can be presented in a matrix as follows:

$$\begin{bmatrix} \overset{0}{Y} \\ \overset{0}{i} \end{bmatrix} = \begin{bmatrix} \varepsilon EDB_Y & \varepsilon EDB_i \\ \eta EDM_Y & \eta EDM_i \end{bmatrix} \begin{bmatrix} Y - Y^e \\ i - i^e \end{bmatrix}$$

Or its equivalent,

$$\begin{bmatrix} \overset{0}{Y} \\ \overset{0}{i} \end{bmatrix} = \begin{bmatrix} \varepsilon(C_Y + I_Y - 1) & \varepsilon(C_i + I_i) \\ \eta m_Y^d & \eta m_i^d \end{bmatrix} \begin{bmatrix} Y - Y^e \\ i - i^e \end{bmatrix} \tag{I}$$

Or in its compact form:

$$\overset{0}{\mathrm{Y}} = \Omega \overset{e}{\mathrm{Y}}$$

Where:

$$\Omega = \begin{bmatrix} \varepsilon(C_Y + I_Y - 1) & \varepsilon(C_i + I_i) \\ \eta m_Y^d & \eta m_i^d \end{bmatrix}$$

The necessary and sufficient conditions for this system to be asymptotically stable, that is, a system where all the movements flow cyclically or non-cyclically towards a stationary equilibrium point, are that the Ω Jacobian matrix trace be negative and its determinant positive.

i) $Tr\Omega < 0$

ii) $Det\Omega > 0$

In this version of the IS-LM, the assumption that the propensity to spend is less than one ($C_Y + I_Y < 1$) ensures that the two stability conditions are met without restrictions.

i) $Tr\Omega = \varepsilon(C_Y + I_Y - 1) + \eta m_i^d < 0$;

ii) $Det\Omega = -\varepsilon\eta[(1 - C_Y - I_Y)m_i^d + m_y^d(C_i + I_i)] > 0$

Stability can also be discussed from the qualitative point of view, by relating the described conditions to the graphic representation of the IS-LM model. For this purpose, we use a phase diagram to represent the goods market and the money market in stationary equilibrium, that is, when output and the interest rate reach equilibrium and stay constant ($\overset{0}{Y} = \overset{0}{i} = 0$).

Stationary equilibrium in the goods market is achieved when output equals demand (excess demand is null), and output remains stable $(\overset{0}{Y} = Y^d - Y = EDB = 0)$. From equation (8), the IS equilibrium equation is obtained in the goods market when $\overset{0}{Y} = 0$.

$$Y = Y^d = C(i, Y) + I(i, Y) \tag{10}$$

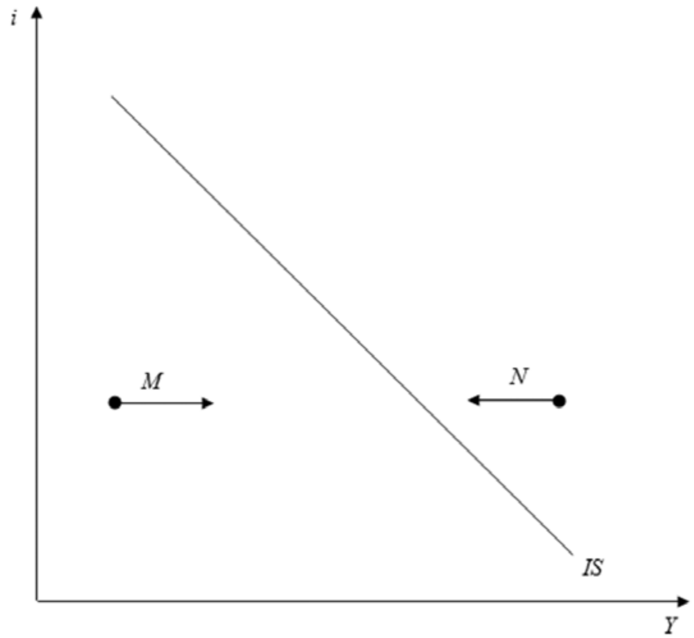
In the (Y, i) space, the IS curve has the usual negative slope, because the propensity to spend is assumed to be less than one, resulting in a positive Keynesian multiplier.

$$\left.\frac{di}{dY}\right|_{IS} = \frac{1}{k(C_i + I_i)} < 0$$

Where $k = \dfrac{1}{1 - C_Y - I_Y} > 0$ is the Keynesian multiplier.

Let us assume that the interest rate slides from any point on the IS in Figure 4, where output equals demand. At this point, let us say point *M* under the IS curve, a lower interest rate increases consumption and investment, which generates excess demand in the goods market. The excess demand in this model, whose dynamics are expressed in equation (8), increases output. This is the meaning of the arrows pointing to the right from point *M*. The same reasoning applies to a point such as *N*, where excess supply reduces output, as shown by the left-pointing arrow that starts at that point.

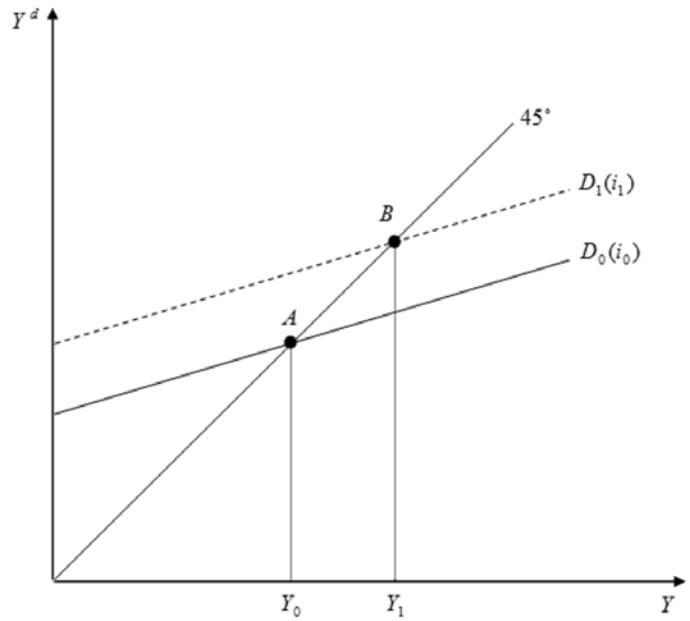
Figure 4



The mechanism allowing for an excess demand to increase production is seen more clearly in the classic 45º diagram such as the one shown in Figure 5. The 45º curve shows the equilibrium points in the goods market, when production (Y) equals demand (Y^d). Curve D is a linear representation of equation (10) with a positive independent component and a slope that is positive, but smaller than one. This reflects a propensity to spend that is lower than one. Curve D has the interest rate as a parameter.

Figure 5 shows that if the interest rate falls from i_o to i_1, curve D shifts upwards to D_1. Excess demand appears at the initial output level Y_0. Such excess demand increases output. The equilibrium point moves from A to B, and production raises to Y_1.

Figure 5



Stationary equilibrium is reached in the money market when supply equals the demand for money, i.e. excess demand is null leading to a stable interest rate ($\overset{0}{i} = m^d - m^s = EDM = 0$). We reach the equilibrium equation in the money market, the LM, when $\overset{0}{i} = 0$ in equation (9).

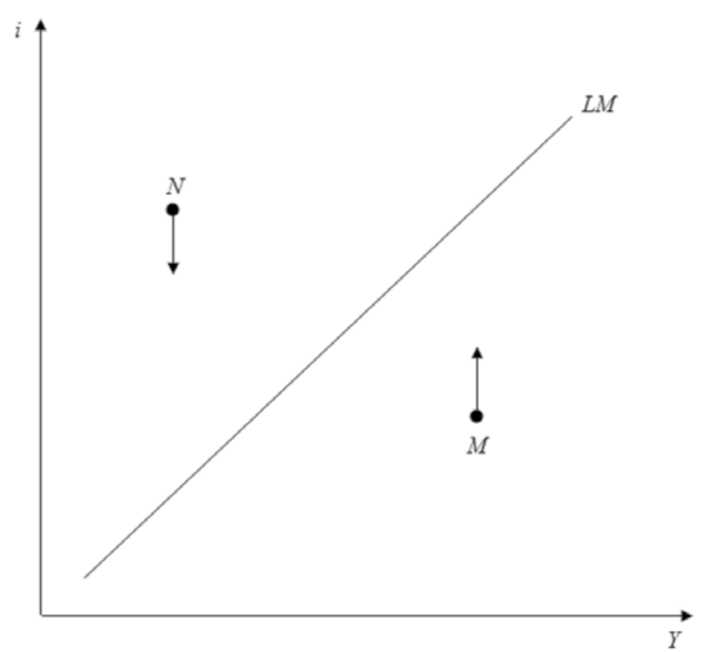
$$\frac{M^s}{P} = m^d(\overset{-}{i}, \overset{+}{Y}) \tag{11}$$

The LM curve slopes positively.

$$\left.\frac{di}{dY}\right|_{LM} = -\frac{m_Y^d}{m_i^d} > 0$$

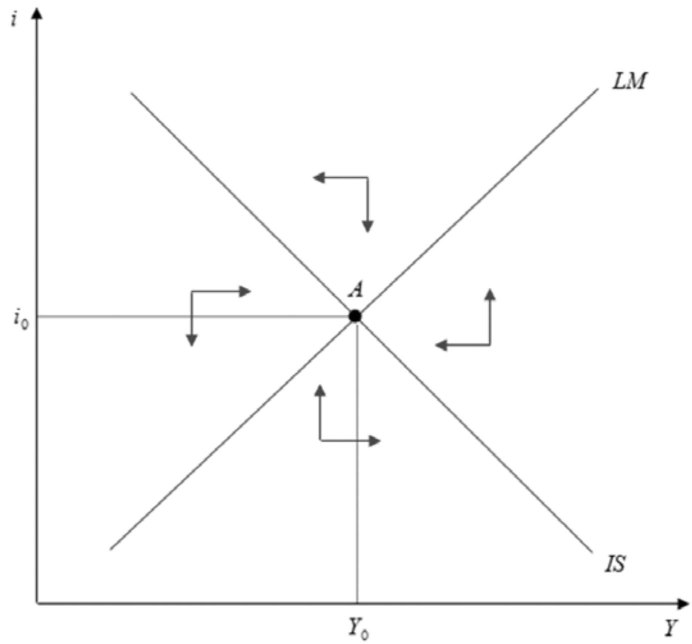
Let us now assume that output increases starting from any point of the LM in Figure 6, where money demand equals supply. Larger output takes us to a point such as *M*, to the right of the LM where the demand for money is also greater, creating excess demand in that market. If we know the dynamics of the money market represented by equation (9), excess demand will shift the interest rate upwards. This explains the upward-pointing arrows starting from point *M*. Symmetrically, to the left of the LM at a point such as *N*, there is an excess money supply, hence the interest rate has to fall, as shown by the downwards-pointing arrow starting at that point.

Figure 6



In a general equilibrium condition we can see the standard IS-LM model is stable, and directional arrows show that we are in the presence of an asymptotically stable equilibrium. At point (Y_0, i_0) the model reaches stationary equilibrium.

Figure 7



Finally, since the second condition for stability, that corresponding to the positive determinant, is equivalent to

$$-\frac{m_Y^d}{m_i^d} > \frac{1}{k(C_i + I_i)}$$

the standard IS-LM model is stable when the LM curve slopes more steeply than the IS.

Starting with Modigliani (1944), the literature has extended to the special case those stability conditions that apply to the standard case. Below, an alternative approach is proposed which delivers the opposite result.

4. IS-LM STABILITY: THE SPECIAL CASE

When the propensity to spend is greater than one ($C_Y + I_Y > 1$), the Keynesian multiplier is negative ($k < 0$) and the IS has positive slope.

$$\left.\frac{di}{dY}\right|_{IS} = \frac{1}{k(C_i + I_i)} > 0$$

If the IS and the LM have positive slopes, we have to determine which slope is greater in order to reach stability.

Our review of the literature found that stability conditions in the special case are reached based on the same equation system (8) and (9). As a result, the same necessary and sufficient conditions for stability are achieved as in the standard case. So, even if $C_Y + I_Y > 1$, the requirement is the same as in the traditional model: the LM must slope more steeply than the IS:

$$-\frac{m_Y^d}{m_i^d} > \frac{1}{k(C_i + I_i)}.$$

This procedure assumes that the dynamics of the goods market, as represented in equation (8), still holds for the special case[4].

$$\overset{0}{Y} = \varepsilon\left[EDB(\overset{-}{Y},\overset{-}{i})\right]; \varepsilon > 0 \tag{8}$$

What does equation (8) tell us? That excess demand in the goods market increases output. Therefore, parameter ε is positive.

If the IS has a positive slope, that is, when $C_Y + I_Y > 1$, is it still true that excess demand in the goods market will increase output?

In the traditional case, excess demand in the goods market disappears when output rises, as represented in equation (8).

Let us assume a fall in the interest rate. A falling interest rate increases investment and consumption, and generates excess demand in the goods market. Excess demand increases output, which, in turn, through its effect on private expenditure, increases demand again. Since demand increases less than output— because the marginal propensity to spend is less than one—the excess demand shrinks. In the movement towards a stationary equilibrium, the excess demand continues to fall until reaching zero, thus reestablishing equilibrium in the goods market.

For a positively sloped IS, the dynamics of adjustment in the goods market is different. We argue that the differential equation reflecting the adjustment dynamics in this market must be reformulated to apply the usual stability conditions. Omission of this aspect has led to the incorrect treatment of stability conditions in the atypical IS-LM model.

Let us assume, as previously, that the interest rate falls, investment and consumption rise, and excess demand is created in the goods market. What adjustment mechanism would be required to restore equilibrium? What must happen to output for such excess demand to be canceled out?

4 Besides, if condition i) holds in the special case, additional restrictions for stability are needed because the sign for $Tr\Omega$ is undetermined. In our review of the literature we found no signs of this issue having been addressed.

If output increases as a response to excess demand, given that the propensity to spend is greater than one, the demand would increase more than production; thus, the excess demand in the goods market, rather than reducing, would rise. The excess demand would continue to grow indefinitely, preventing the system from reaching a steady state. This dynamic is clearly explosive.

For the process to be convergent output must contract rather than increase. As output shrinks, demand for goods falls more than output because the propensity to spend is greater than one, thus reducing excess demand. That is, in this special case, excess demand in the goods market reduces output. Or, likewise, if the Keynesian multiplier is negative, an increase in demand reduces, and does not increase, output.

We propose an alternative adjustment dynamics equation in the goods market by reproducing the mechanism we have just described. In the differential equation for the goods market, output increases should be related to excess supply in the market ($EOB = Y - Y^d$), not to excess demand. The equation that adequately reflects the new adjustment dynamics of the goods market can be written as:

$$\overset{0}{Y} = \varepsilon(EOB); \varepsilon > 0 \tag{12}$$

Excess supply in the goods market is defined by:

$$EOB = Y - Y^d = EOB(\overset{-}{Y}, \overset{+}{i}); \; EOB_Y = 1 - C_Y - I_Y < 0; EOB_i = -(C_i + I_i) > 0 \tag{13}$$

Thus, the new adjustment dynamics in the goods market, which replaces equation (8), is now written as:

$$\overset{0}{Y} = \varepsilon\left[EOB(\overset{-}{Y}, \overset{+}{i})\right]; \varepsilon > 0; \tag{14}$$

This formulation is consistent with Gandolfo's mathematical requirement (1996, p. 328) noted above.

Since there is no change in the money market, the new differential equation system in the atypical case is comprised of:

$$\overset{0}{Y} = \varepsilon\left[EOB(\overset{-}{Y}, \overset{+}{i})\right]; \varepsilon > 0; \tag{14}$$

$$\overset{0}{i} = \eta\left[EDM(\overset{+}{Y}, \overset{-}{i})\right]; \eta > 0 \tag{9}$$

As before, linearizing the equation system (14) and (9) using Taylor's expansion yields the following:

$$\begin{bmatrix} \overset{0}{Y} \\ \overset{0}{i} \end{bmatrix} = \begin{bmatrix} \varepsilon(1-C_Y-I_Y) & -\varepsilon(C_i+I_i) \\ \eta m_Y^d & \eta m_i^d \end{bmatrix} \begin{bmatrix} Y-Y^e \\ i-i^e \end{bmatrix} \tag{II}$$

Or in its compact form:

$$\overset{0}{\mathrm{Y}} = \Omega_1 \overset{e}{\mathrm{Y}}$$

Where:

$$\Omega_1 = \begin{bmatrix} \varepsilon(1-C_Y-I_Y) & -\varepsilon(C_i+I_i) \\ \eta m_Y^d & \eta m_i^d \end{bmatrix}$$

The necessary and sufficient conditions for this differential equation system to be stable are, as before, for the matrix Ω_1 trace to be negative and its determinant positive.

iv) $Tr\Omega_1 = \varepsilon(1-C_Y-I_Y)+\eta m_i^d < 0;$

v) $Det\Omega_1 = \varepsilon\eta\left[(1-C_Y-I_Y)m_i^d + m_Y^d(C_i+I_i)\right] > 0$

The first condition is met with no restrictions. To meet the second condition, we must have:

$$\frac{1}{k(C_i+I_i)} > -\frac{m_Y^d}{m_i^d},$$

Otherwise put, when the propensity to spend is greater than one and the Keynesian multiplier is negative, for IS-LM to be stable, the IS must be steeper than the LM:

$$\left.\frac{di}{dY}\right|_{IS} > \left.\frac{di}{dY}\right|_{LM}$$

We reach the same conclusion using the phase diagram system. Since no change has occurred in the money market, let us focus our attention on the positively sloping IS.

Let us assume a fall in interest rates starting at some point of the IS in Figure 8, where output equals and demand. At point *M*, under the IS curve, the lower interest rate increases investment and consumption, creating excess demand in the goods market. In the framework of this model, such excess demand in the goods market - the dynamics of which are expressed in equation (14) - contracts, and <u>does not increase</u>, output, as is erroneously assumed. This explains the arrow to the left that starts from point *M*.

By the same reasoning, at a point such as *N* there is excess supply that increases output, as symbolized by the right-pointing arrow in Figure 8.

Figure 8

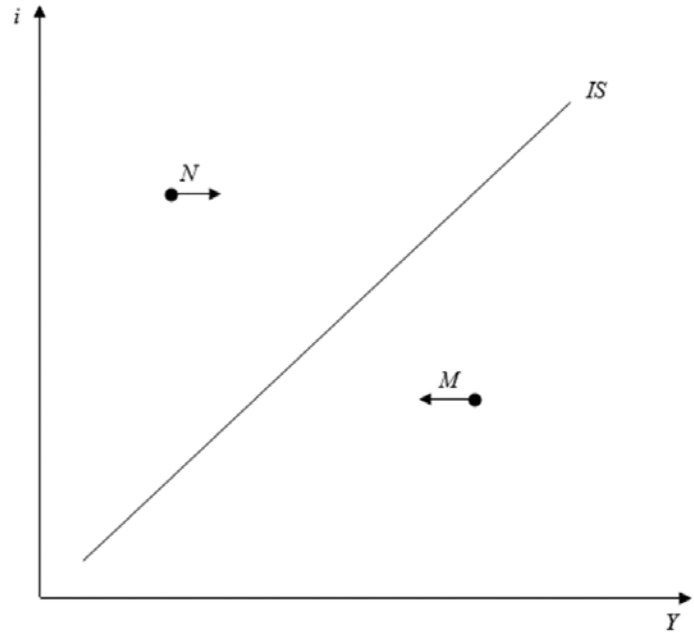
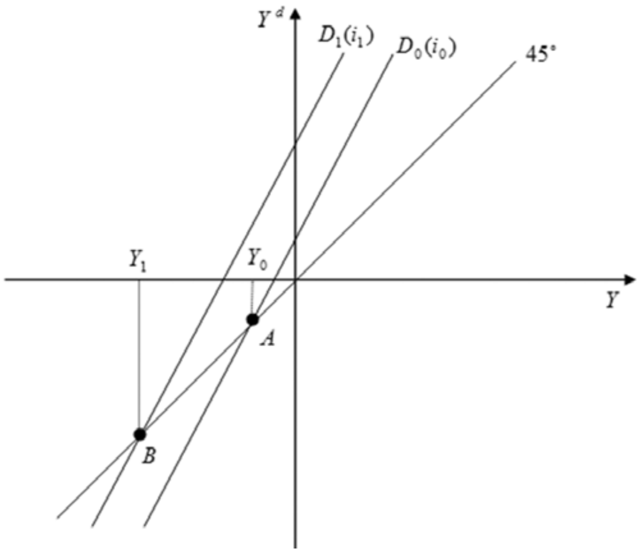


Figure 9 replicates the 45º diagram for the atypical case. As before, curve *D* is the linear representation of equation (10). The difference is now that its slope is greater than one because the propensity to spend is greater than one. In these conditions, equilibrium can only occur in the quadrant where the value of output is negative. We will return to this point in section 5 below.

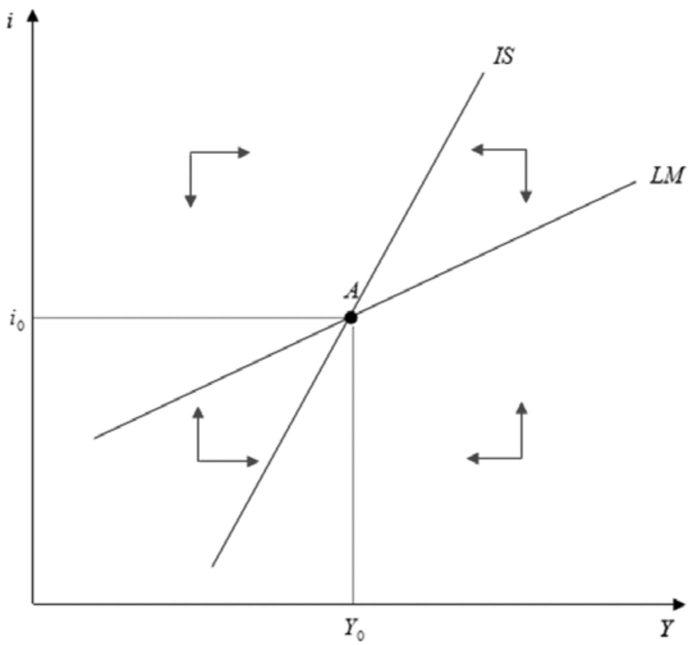
Let us simulate, as before, the interest rate falls from i_0 to i_1. In Figure 9, if the interest rate falls, curve *D* moves upwards to D_1. At the initial production level Y_0, there is an excess demand in the goods market. This excess demand reduces output; it does not increase it. The equilibrium point shifts from *A* to *B*, and output falls toY_1.

Figure 9



In general equilibrium, Figure 10 combines figures 1 and 2 in a single diagram, showing that the IS-LM model is stable when the IS is steeper than the LM[5].

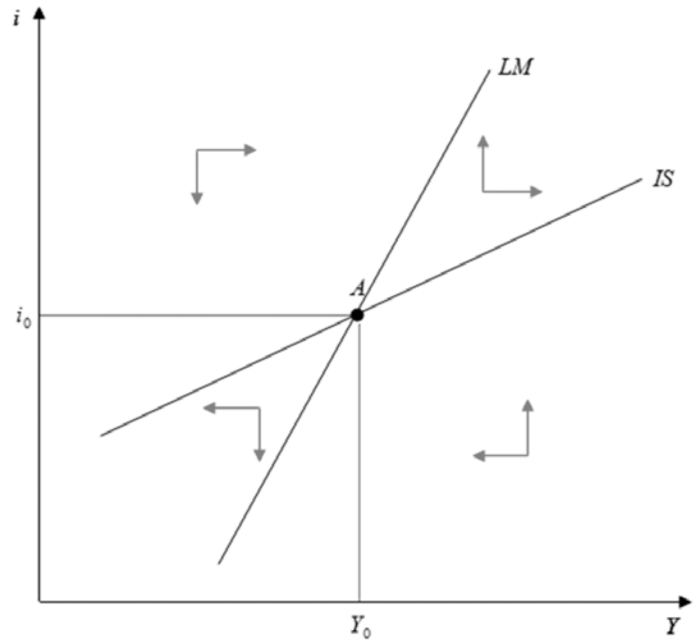
Figure 10



[5] In section 5 we will show that in the special case of a linear IS-LM model, the equilibrium values for output and the interest rate are negative.

In Figure 11 the phase diagram shows that when the LM is steeper than the IS, the model is unstable, of the saddle point type.

Figure 11



5. STABILITY, INSTABILITY AND EXPANSIVE MONETARY POLICY

An additional argument in discussing instability conditions is provided by changing an exogenous variable and evaluating how the endogenous variables will adjust. In stable models, after changing an exogenous variable, these variables temporarily depart from their initial equilibrium values, but then change until they reach a new stationary equilibrium. When models are unstable, endogenous variables move away from the initial equilibrium and never reach a new stationary equilibrium.

In Dernburg and McDougall (1976) the effects of an expansive monetary policy are illustrated within the context of a positively sloped IS. Their presentation – which assumes stability when the LM is steeper than the IS – is useful to provide evidence of the error of considering that, in this model, excess demand in the goods market leads to increased output.

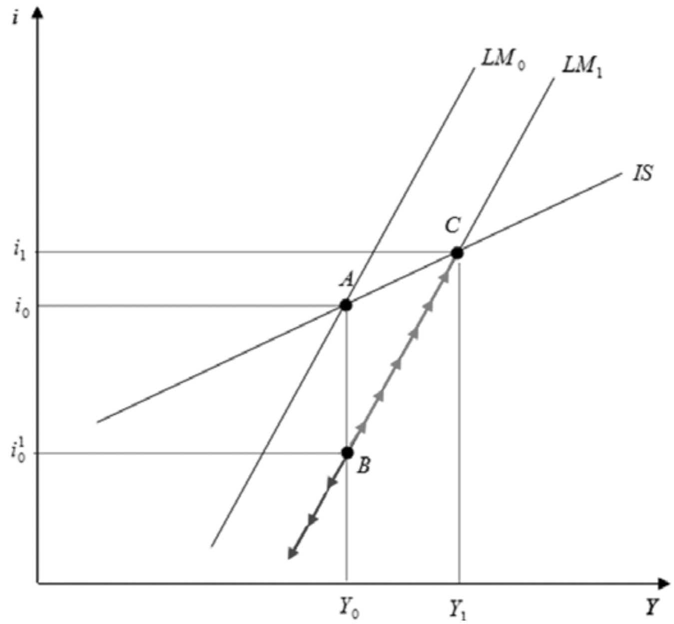
Dernburg and McDougall (1976) repeat the exercise with a diagram such as that shown in Figure 12, assuming that the money market is always in equilibrium and the goods market may be temporarily in disequilibrium.

In Figure 12, an expansive monetary policy (an increase in M^s), shifts the LM to the right. According to Dernburg and McDougall:

> The shift in the LM curve causes the interest rate to fall immediately to i_0^1. This again means that intended investment exceeds saving and that income must therefore rise. But the rise in income stimulates further investment because of our assumption that investment is a function of the level of profits and income. Consequently, the original monetary disturbance causes income to rise; this cause additional investment to be induced, and this, in turn, causes income to rise still further.
>
> Will income continue to rise indefinitely, or will a new equilibrium point be found? In the present case the rise in income causes the interest rate to rise and to dampen investment more rapidly than the rise in income stimulates further investment. In other words the rate of interest that keeps the money market in equilibrium rises more rapidly than the rate of interest that keeps the product market in equilibrium. Consequently, a new stable equilibrium point will be reached at Y_1 and i_1. The path of adjustment again follows the arrows upward along the LM curve" (Dernburg and McDougall 1976, p. 244)

Figure 12 shows the above mentioned authors' graph and argument. The red arrows highlight their proposed adjustment dynamics.

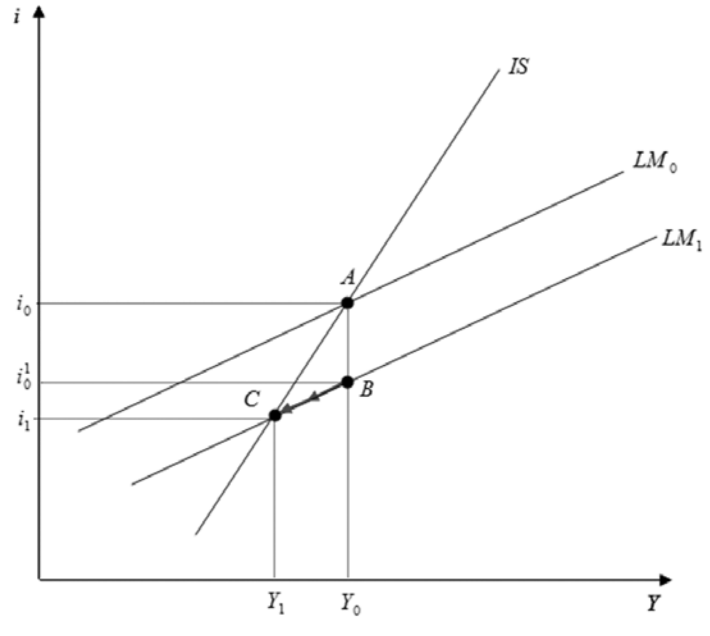
Figure 12



What is the mistake in this reasoning? Positing that when the interest rate falls to point *B*, the resulting excess demand increases output. In this atypical IS-LM model excess demand reduces output, rather than increasing it. Therefore, since at point *B* there is excess demand in the goods market, output should fall and the blue arrows should point to the left, not to the right. Therefore, the model is unstable and the system does not reach a stationary equilibrium.

Figure 13 shows the stable case, when the IS is steeper than the LM. As before, the expansive monetary policy shifts the LM to the right, and short term equilibrium is reached at point *B*, with a lower interest rate and the same production level. As in *B* the lower interest rate has created excess demand in the goods market, output must fall to restore equilibrium in that market. As output falls, so too does the demand for money, and the interest rate slips as well. Downward adjustment continues (following the arrows) along the new LM curve until it reaches (Y_1, i_1). At this point, a new stationary equilibrium is reached.

Figure 13



The fact that an expansive monetary policy should reduce instead of increasing the interest rate, as in the unstable case shown Figure 11, is consistent with Samuelson's observation seven decades ago. By simulating the effects of an expansive monetary policy in different scenarios, including a propensity to spend of greater than one, he established that:

> ..the only theorem which remains true under all circumstances is that an increase in the amount of money must lower interest rates if equilibrium is stable. (Samuelson, 1941, p. 120).

If the LM were steeper than the IS, as in Figure 12 - which reproduces Dernburg's and MacDougall's arguments (1976) - an expansive monetary policy would increase the interest rate.

6. STABILITY, NEGATIVE KEYNESIAN MULTIPLIER AND THE RELEVANCE OF THE ATYPICAL CASE

We have argued that in the atypical case, IS-LM model is stable when the IS is steeper than the LM. In this section we will show that in this special stable case, the model yields results of no useful economic significance.

Firstly, in the model's linear version, output and interest rate equilibrium values are negative. Secondly, since the Keynesian multiplier is negative, expanding demand results in falling output.

We have adopted the linear version of the IS-LM model to illustrate how to obtain analytical equilibrium values for output and the interest rate. Equilibrium in the goods and money markets is given by:

$$Y = Y^d = C_0 + a_0 Y - a_1 i + I_0 + a_2 Y - a_3 i \tag{15}$$

$$M^s - P = m^s = m^d = b_0 Y - b_1 i \tag{16}$$

Where $C_0 + I_0 = A_{o_o}$ is autonomous private expenditure, M^s is the nominal money supply, P is the price level, and all the parameters are positive.

The IS and LM equations are drawn from these formulae, respectively:

$$i = \frac{A_o}{a_1 + a_3} - \frac{1}{k(a_1 + a_3)} Y \tag{17}$$

$$i = -\frac{M^s - P}{b_1} + \frac{b_o}{b_1} Y \tag{18}$$

where $k = \dfrac{1}{1 - a_0 - a_2} < 0$ is the Keynesian multiplier.

Equilibrium in the goods market can also be expressed in such a way that the consequences of having a negative Keynesian multiplier will be more clearly apparent. In this case, as shown in equation (19), greater demand – prompted by rising autonomous expenditure or a sliding interest rate – leads to lower output.

$$Y = k[A_o - (a_1 + a_3)i] \tag{19}$$

The IS and LM slopes are given by:

$$\left.\frac{di}{dY}\right|_{IS} = -\frac{1}{k(a_1 + a_3)} > 0; \quad \left.\frac{di}{dY}\right|_{LM} = \frac{b_o}{b_1} > 0$$

To meet the second stability[6] condition, the IS must be steeper than the LM:

$$-\frac{1}{k(a_1+a_3)} > \frac{b_0}{b_1}$$

That is,

$$b_1 + k(a_1 + a_3)b_0 > 0$$

Furthermore, the stationary equilibrium values for output and the interest rate are obtained from equations (17) and (18).

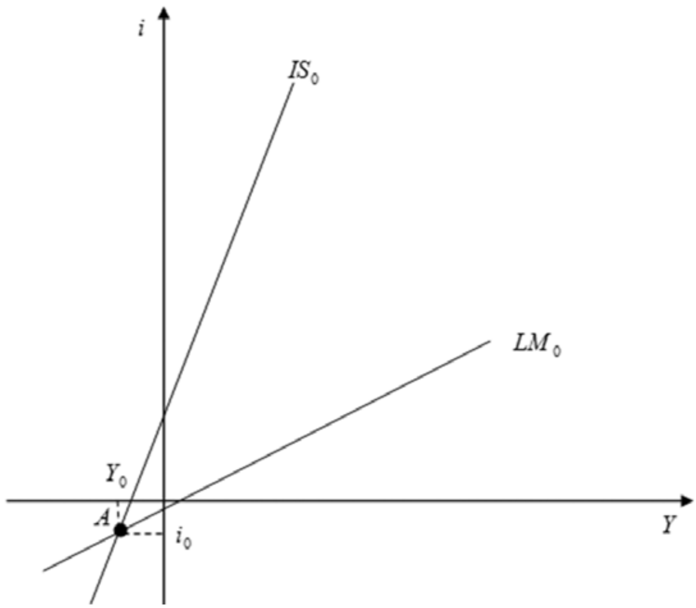
$$Y^e = \frac{kb_1}{b_1 + k(a_1+a_3)b_0} A_0 + \frac{k(a_1+a_3)}{b_1 + k(a_1+a_3)b_0}(M^s - P) \tag{20}$$

$$i^e = \frac{b_0 k}{b_1 + k(a_1+a_3)b_0} A_0 - \frac{1}{b_1 + k(a_1+a_3)b_0}(M^s - P) \tag{21}$$

Given the stability condition $b_1 + k(a_1 + a_3)b_0 > 0$, the stationary equilibrium values of output and the interest rate are evidently negative, which makes no economic sense.

Figure 14, where equations (17) and (18) are represented with the IS slope larger than the LM slope, reproduces this result. Stationary equilibrium is reached in the quadrant where interest rate and product values are negative.

Figure 14



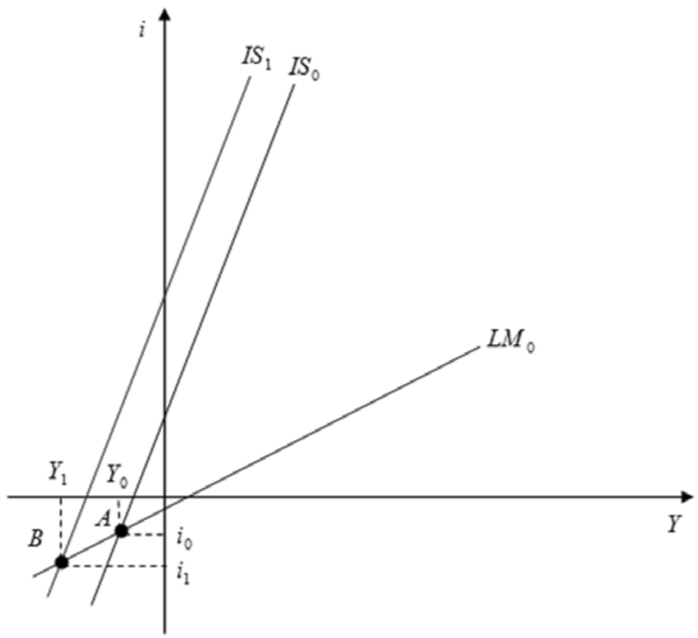
[6] The trace condition is met without restriction.

This IS-LM model configuration leads us to a situation where exogenous shocks produce analytically strange outcomes.

For example, in this model, an autonomous expansion of consumption or investment – or, if we introduce government to the model, expanding government expenditures – would lead to falling output and interest rates. This is because greater expenditure generates excess demand in the goods market, thus reducing output. Falling output reduces demand for money in the money market, thus bringing the interest rate down.

In Figure 15, in view of a larger autonomous expenditure, the IS curve shifts upwards. In the new point of intersection with curve LM, which has not shifted, both output and the interest rate are smaller.

Figure 15



7. A SOLUTION TO THE IMPASSE: THE CLASSICAL MECHANISM

Although in strictly mathematical terms our results are correct, in theoretical and empirical terms the notion that firms reduce production to meet a rise in demand for their goods, or that the equilibrium values of output and the interest rate are negative, render the results absurd.

In formal terms, the only way to justify the position of Modigliani and later economists would be to abandon the Keynesian model, which assumes that the adjustment in the goods market occurs through changes in production and the adjustment in the money market through changes in the interest rate. If, as in the Classical case – where real money demand does not depend on the rate of interest – production is determined in the money market and the interest rate in the goods market, it can be shown, with the same formal arguments used, that the stable case is that where the LM is steeper than the IS. This definition of Classical model corresponds to Hicks (1937).

If we adopt the Classical adjustment mechanism, the dynamics of the determination of the interest rate and output would be given by:

$$\overset{0}{r} = g\left[EDB(\overset{+}{Y}, \overset{-}{i}) \right]; g > 0; \tag{22}$$

$$\overset{0}{Y} = h\left[EOM(\overset{-}{Y}, \overset{+}{i}) \right]; h > 0 \tag{23}$$

Equation (22) indicates that the interest rate rises when there is excess demand in the goods market and equation (24) shows us that production rises when there is excess supply in the money market.

Linearizing the equation system (22) and (23) yields the following:

$$\begin{bmatrix} \overset{0}{r} \\ \overset{0}{Y} \end{bmatrix} = \begin{bmatrix} g(C_i + I_i) & -g(1 - C_Y - I_Y) \\ -hm_i^d & -hm_Y^d \end{bmatrix} \begin{bmatrix} i - i^e \\ Y - Y^e \end{bmatrix}$$

Or in its compact form:

$$\overset{0}{\mathrm{Y}} = \Omega_2 \overset{e}{\mathrm{Y}}$$

Where:

$$\Omega_2 = \begin{bmatrix} g(C_i + I_i) & -g(1 - C_Y - I_Y) \\ -hm_i^d & -hm_Y^d \end{bmatrix}$$

The necessary and sufficient conditions for this differential equation system to be stable are:

i) $Tr\Omega_2 = g(C_i + I_i) - hm_Y^d < 0;$

ii) $Det\Omega_2 = -gh\left[(C_i + I_i)m_Y^d + m_i^d(1 - C_Y - I_Y) \right] > 0$

The first condition is met without problem. To meet the second condition, we must have,

$$\frac{1}{k(C_i + I_i)} < -\frac{m_Y^d}{m_i^d},$$

that is, when the propensity to spend is greater than one but the adjustment mechanism is Classical, for IS-LM to be stable, the LM must be steeper than the IS:

$$\left.\frac{di}{dY}\right|_{LM} > \left.\frac{di}{dY}\right|_{IS}$$

However, none of the studies reviewed departed from the Keynesian adjustment mechanism when discussing the stabilities of the IS-LM model in the special case.

8. CONCLUSIONS

This paper argues that in the special case where IS is positively sloped, for the IS-LM model to be stable, the IS must be steeper than the LM. This challenges the literature that, starting with Modigliani (1944), holds that stability is reached when the LM is steeper than the IS.

Only Samuelson (1941), by simulating the effects of an expansive monetary policy in different scenarios, including the case of a propensity to spend of greater than one, proposes a result that is consistent with ours.

However, in this special stable case, the model yields results of no useful economic significance. Firstly, in the model's linear version, the equilibrium values for output and the interest rate are negative. Secondly, since the Keynesian multiplier is negative, expanding demand results in falling output.

In this paper one solution to the problem identified was raised. That solution is to abandon the Keynesian adjustment mechanism and replace it with the Classical mechanism where the interest rate is determined in the goods market and production in the money market. In this case, the IS-LM model is stable when the LM is steeper than the IS.

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