Metal Prices and International Market Risk in the Peruvian Stock Market*

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ABSTRACT

In this paper we use the conditional Value at Risk (CoVaR) and CoVaR variation (ΔCoVaR) proposed by Adrian and Brunnermeier (2008, 2011, 2016) to estimate the Peruvian stock market risk (through the IGBVL) conditioned on the international financial market (given that the S&P500) and conditioned on three of the main commodities exported by Peru: copper, silver and gold. Moreover, the CoVaR measures are compared with the VaR of the IGBVL to understand the differences using conditional and unconditional risk measure estimators. The results show that both CoVaR and ΔCoVaR are useful indicators to measure the Peruvian stock market risk.

Keywords: Commodities, copula, CoVaR, S&P500, VaR

JEL classifications: C5, G01, G10, G18, G20, G28, G32, G38

Precio internacional de los metales y riesgo de mercado en la Bolsa de Valores de Lima

RESUMEN

En este trabajo utilizamos el Valor en Riesgo condicional (CoVaR) y la variación CoVaR (ΔCoVaR) propuestos por Adrian and Brunnermeier (2008, 2011, 2016) para estimar el riesgo bursátil peruano (a través del IGBVL) condicionado en el mercado internacional (dado por el índice S&P500) y condicionado en tres de los principales *comodities* exportados por el Perú: cobre, plata y oro. Además, las medidas CoVaR son comparadas con el VaR del IGBVL para entender las diferencias al utilizar medidas de riesgo condicionales e incondicionales. Los resultados muestran que ambas medidas CoVaR and ΔCoVaR constituyen indicadores útiles para estimar el riesgo bursátil peruano.

Palabras clave: Comodities, copula, CoVaR, S&P500, VaR

JEL: C5, G01, G10, G18, G20, G28, G32, G38

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1. INTRODUCTION

In times of financial crisis, the losses of financial institutions in distress tend to spread to the rest of the financial system. During these times of stress, the correlation between financial variables and the relationships between markets or institutions also tend to increase (Acharya, 2009).

Usually, traditional risk measures such as the (unconditional) Value at Risk (VaR) have been used to estimate the individual risk of each entity, but without explicitly considering the co-dependency that may exist in relation to the risk of other entities and other markets. This limitation of unconditional VaR methods has become extremely important in the light of the financial crisis of the late 2000s. Since this crisis, a growing consensus has emerged among policy makers, risk managers and academic researchers about the importance of adopting different approaches to measure risk and mitigate the risks inherent to the financial system as a whole. For this reason, systemic risk and its management have become a key regulatory issue. Measures to mitigate this risk constitute macroprudential regulation, which addresses the financial system as a whole, giving priority to the interrelationships between its components and resultant effects on the rest of the economy (Gauthier et al., 2010).

In the literature we found several ways to measure systemic risk. Examples include the work of Acharya et al. (2010), Bae et al. (2003), Chan-Lau (2010), Gauthier et al. (2010), Huang et al. (2010) and Brownless and Engle (2012). Of these, one of the most important methodologies is that proposed by Adrian and Brunnermeier (2008), the conditional VaR[1]: CoVaR. The $CoVaR^{i/j}$ measures the VaR of institution i given that institution j is in financial distress, that is, when institution j has the same return as its VaR. In addition, to measure the marginal contribution to the risk to institution i by institution j, Adrian and Brunnermeier (2008) propose the CoVaR variation: ΔCoVaR, which is defined as the difference between the CoVaR in financial distress and the CoVaR measured in normal situations.

The advantage of CoVaR over the traditional VaR lies in the fact that the risk of institution i is estimated by accounting for the transmission risk from institution j. This is the main reason why CoVaR methods have been applied in different parts of the world. For example, Rungporn and Phurichai (2010) use it in Thailand, while Danielsson et al. (2011) do so for the USA. In the particular case of Latin America, examples include Arias et al. (2010) in Colombia, Almeida et al. (2012) in Brazil, and Castelao et al. (2012) in Uruguay.

In the case of Peru, Espino and Rabanal (2011) measure the systemic risk of the banking sector using the methodology of Chan-Lau (2012). However, as far as we know,

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[1] As mentioned by Adrian and Brunnermeier (2008, 2011, 2016), "co" also refers to co-movement or contagion.

there are no references concerning the application of the CoVaR methodology of Adrian and Brunnermeier (2008, 2011, 2016) for this or other sectors of the Peruvian economy.

Although originally proposed as a measure of systemic risk, the CoVaR concept has been applied to several financial areas; see Wong and Fong (2011) and the references therein. Thus, we believe that the CoVaR is an interesting measure of risk capital, which can be applied not only to institutions but also to markets in general to measure linkages that may exist in between them. Thus, in this paper we propose to use the CoVaR method to estimate the Peruvian stock market risk. Specifically, we want to evaluate and quantify how the Peruvian stock market risk (measured through the general index of the Lima Stock Exchange: the IGBVL) depends on the international stock market (as proxied by the S&P500) and the international prices of copper, gold and silver. We chose these metals due to the relatively significant weights that the mining companies that produce them have in the composition of the IGBVL, and because these metals constitute three of Peru's major export commodities. For this purpose, we collected a sample of IGBVL, S&P500, copper, gold and silver prices in the period 02/01/2004 to 31/12/2013.

The main objective of this study is to critically evaluate the CoVaR method in terms of the estimation of Peruvian stock market risk, recording the advantages and disadvantages compared to traditional risk measures such as the (unconditional) VaR. Thus, this research extends the previous studies of Zevallos (2008), del Carpio and Zevallos (2010), and Calderon and Rodriguez (2014) in which Peruvian stock market risk is estimated through unconditional VaR methods without explicitly considering the risk transmission from other markets.

To calculate the CoVaR we adopt the definition of Girardi and Ergun (2013) and Mainik and Schaanning (2014). An additional contribution of this work is its proposal of a way to calculate the CoVaR in normal periods. Consequently, the calculation of CoVaR and ΔCoVaR differs from that of Adrian and Brunnermeier (2008, 2011).

In this work, both CoVaR and ΔCoVaR are considered as time-dependent, *i*.e., we consider that these measures evolve in time. Two estimates are calculated based on the available information. First, since the objective is to estimate the marginal contribution to risk, the ΔCoVaR at time t will be calculated using the information until time t. Second, facing a realistic risk management situation in which we only have past information, we calculate the one-step-ahead CoVaR prediction. Thus, using the information up to time $t-1$ we calculate the CoVaR prediction at time t.

In the literature we find several methods of estimating the CoVaR. Adrian and Brunnermeier (2011) use quantile regression and compare it with multivariate GARCH. Chao et al. (2014) estimate the CoVaR through semiparametric regression quantile models. Girardi and Ergun (2013) employ multivariate GARCH models but using a different definition of CoVaR. Hakwa et al. (2012) and Chen and Khashanah (2014) among others, use copula methods.

Copula methods are a versatile means of capturing the dependence on financial series, see for example Cherubini et al. (2004) and Patton (2012). For this reason, here we estimate the CoVaR and ΔCoVaR via simulated copulas (unlike Hakwa et al. [2012] and Chen and Khashanah [2014]). Specifically, we simulate bivariate series in which the marginal models follow heteroscedastic conditional variance models and the dependence between shocks is governed by a copula model. This simulation method has been used by, among others, Dias and Embrechts (2003), Patton (2006) and Palaro and Hotta (2006). In particular, the latter authors apply the methodology to estimate unconditional VaR but not CoVaR.

The remainder of this paper is organized as follows. In Section 2 we briefly present the CoVaR and ΔCoVaR measures and their estimation methodology. In Section 3 we present the empirical analysis based on the method CoVaR. Finally, in Section 4 we conclude and provide some future research recommendations.

2. METHODS

In this section, we start by presenting the definitions of the CoVaR and ΔCoVaR risk measures. Then, we describe the procedure adopted to estimate these measures.

2.1. CoVaR and ΔCoVaR

Let $\{x_t\}$ and $\{y_t\}$ be two sets of time series returns, where $t = 1, \ldots\ldots, T$. As to the definitions of the considered risk measures, the VaR of $\{y_t\}$ at confidence level q, denoted by VaR^y (q), is defined as

$$P(y_t < VaR^y(q)) = q \tag{1}$$

Note that this is an unconditional measure because the VaR is calculated using only the y series; that is, using only the unconditional distribution of y_t. In order to obtain a conditional risk measure, Adrian and Brunnermeier (2011) propose using the VaR of y_t conditioned on some event $C(x)$ of x_t .This measure, called CoVaR and denoted by $\text{CoVaR}^{y/C(x)}(q)$ is formally defined as

$$P(y_t \leq CoVaR^{(y|C(x))}(q)|C(x_t)) = q \tag{2}$$

To estimate the risk of y conditional on x under *financial distress*, Adrian and Brunnermeier (2011) propose $C(x_t) = [x_t = VaR^x(q)]$. Furthermore, in order to measure the marginal contribution to the risk of y caused by x, Adrian and Brunnermeier (2011) propose the CoVaR *variation*, denoted by ΔCoVaR, which is defined as the difference between the CoVaR when x is under *financial distress*, and the CoVaR when x is in *normal periods or in some benchmark state*. The CoVaR in normal periods, these authors suggest, is calculated by setting $C(x_t) = \text{Median}(x_t)$.

Instead of conditioning exactly on $VaR^x(q)$, Girardi and Ergun (2013) and Mainik and Schaanning (2014) propose conditioning on equal or less $VaR^x(q)$ values. In this way, the risk measure can account for even more extreme events. In this paper we follow this approach and, in addition, we permit two confidence levels, one for x and one for y. Therefore, the definition of CoVaR we adopt is,

$$P(y_t \leq CoVaR^{(y|x)}\,(q, p)|x_t \leq VaR^x\,(p)) = q \tag{3}$$

In this sense, for small values of p and q, $CoVaR^{(y|x)}$ (q, p) measures the risk of y given that x is in a situation under financial distress, taking into account the dependence (or the linkages) between the individual risks of x and y. Moreover, in this paper we propose a different way to calculate the CoVaR in normal situations. Unlike Adrian and Brunnermeier (2011) and motivated by Girardi and Ergun (2013)[2], we propose to measure the CoVaR in normal situations, denoted by $CoVaR^{(y|x)}$ $(q, *)$, as

$$P(y_t \leq CoVaR^{(y|x)}\,(q, *)|x_t \in (Q_{1x}, Q_{3,x})) = q \tag{4}$$

where Q_{1x} and Q_{3x} are the first and third quartiles of the distribution of x, respectively. Then, the ΔCoVaR is calculated as

$$\Delta CoVar^{(y|x)} = CoVar^{(y|x)}\,(q, p) - CoVar^{(y|x)}\,(q, *) \tag{5}$$

2.2. Estimation of CoVaR and ΔCoVaR

As mentioned in the introduction, we find several methods to estimate the CoVaR in the literature. Here, we use copula methods to calculate the CoVaR via simulation. Specifically, we simulate bivariate series where the marginal univariate models follow heteroscedastic conditional variance models, and the dependence between the shocks is governed by a copula model. Among others, simulation methods have been used by Dias and Embrechts (2003), Patton (2006), and Palaro and Hotta (2006), who apply the method in estimating the unconditional VaR but not the CoVaR.

Consider two sets of time series returns $\{x_1, \ldots., x_T\}$ and $\{y_1, \ldots., y_T\}$. To simplify subsequent exposition, we denote by x_t and $r_{1,t}$ and y_t and $r_{2,t}$ Assume that each series presents conditional mean which evolves as an autoregressive model of order one, and conditional variance according to an APARCH model (Ding et al., 1993). Specifically, for $i = 1, 2$ and $t = 1, \ldots\ldots, T$,

$$r(i, t) = \mu_{i,t} + \in_{i,t} \tag{6}$$

$$\mu_{i,t} = c_i + \phi r_{i,t-1} \tag{7}$$

[2] In this study, the authors use the VaR of y_t conditioned on the event $[x_t \in (\mu_x - \sigma_x, \mu_x + \sigma_x)]$ where μ_x and σ_x are the mean and standard deviation of the distribution of x_t, respectively.

$$\epsilon_{i,t} = \sigma_{i,t}\eta_{i,t} \tag{8}$$

$$\sigma_{i,t}^{\delta_i} = \omega_i + \alpha_i\left(\left|\epsilon_{i,t-1}\right| - \gamma_i\,\epsilon_{i,t-1}\right)^{\delta_i} + \beta_i\sigma_{i,t-1}^{\delta_i} \tag{9}$$

where δ_i is a positive real number and parameters ω_i, α_i , β_i and γ_i satisfy certain conditions so that $\sigma_{i,t}$ (the volatility) is positive. Moreover, for each $i, \{\eta_{i,t}\}$ is a sequence of independent and identically distributed random variables with generalized exponential distribution (GED) with mean 0, variance 1 and shape ν_i parameter (see Nelson, 1991).

We chose model (6)-(9) because it is versatile enough to reproduce many of the empirical characteristics of financial time series returns: serial correlation in levels (through the parameter ϕ), volatility clustering, heavy tails (through the GED distribution), the leverage effect (through the parameter γ) and evolution of powers of volatility (through the parameter δ).

To reproduce the dependence between the two sets of returns, we assume that ($\eta_t = (\eta_{1,t}, \eta_{2,t}$) follows a *copula* model. Consider that we have a bivariate vector (η_1, η_2) without temporal dimension t. According to the bivariate version of Sklar's theorem (Sklar, 1959), for continuous variables there is a unique copula function C such that the joint distribution function of (η_1, η_2) denoted by F satisfies

$$F(\eta_1,\eta_2) = C(F_1(\eta_1), F_2(\eta_2)) \tag{10}$$

where F_1 and F_2 are the marginal distribution functions of η_1 and η_2, respectively. Additionally, this theorem guarantees that

$$C(u_1,u_2) = F\left(F_1^{-1}(u_1), F_2^{-1}(u_2)\right) \tag{11}$$

where $u_i = F_i(\eta_i)$ and F_i^{-1} is the inverse distribution function of η_i for $i = 1,2$. Therefore, the *copula density* is

$$c(u_1,u_2;\theta) = \frac{\partial^2 C(u_1,u_2)}{\partial u_1 \partial u_2} \tag{12}$$

where q is a vector which contains the copula parameters. Then, as a consequence of (10), the density of (η_1, η_2), f satisfies

$$f(\eta_1, \eta_2) = c(u_1, u_2; \theta) f_1(\eta_1), f_2(\eta_2) \tag{13}$$

where f_i is the density of η_i, $i = 1,2$. The versatility of the copula model in reproducing multivariate distributions is evident in (13): a bivariate density is the result of the product of two univariate marginal densities and a copula density which captures the dependence between the variables.

To capture the temporal evolution of the dependence between sets, here we use the dynamic version of Sklar's theorem given by Patton (2006), in which the dependence between variables is conditional on the past information denoted by $\mathcal{F}_t$. That is, $F(\eta_1,\eta_2 \mid \mathcal{F}_t) = C\left(F_1(\eta_1 \mid \mathcal{F}_t), F_2(\eta_2 \mid \mathcal{F}_t)\right)$.

In this work we consider two copulas usually employed in empirical applications: the *t*-Student copula defined as,

$$C(u_1,u_2;\nu,\rho) = \int_{-\infty}^{t_\nu^{-1}(u_1)} \int_{-\infty}^{t_\nu^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1+\frac{s^2-2\rho st+t^2}{\nu(1-\rho^2)}\right)^{-(\nu+2)/2} \tag{14}$$

where t_ν^{-1} is the inverse function of the *t*-Student with ν degrees of freedom, and the Joe-Clayton copula defined as

$$C(u_1,u_2;\tau_L,\tau_U) = 1-\left(\left\{\left[1-(1-u_1)^\kappa\right]^{-\gamma} + \left[1-(1-u_2)^\kappa\right]^{-\gamma} - 1\right\}^{1/\gamma}\right)^{1/\kappa} \tag{15}$$

where $\kappa = 1/\log_2(2-\tau_U)$, $\gamma = -1/\log_2(\tau_L)$ with parameters $\tau_L \in (0,1)$ and $\tau_U \in (0,1)$.

Next, we describe how, given the information up to time t and given the confidence levels p and q, we estimate the $CoVar^{(y|x)}(q, p)$ at time $t+1$. That is, how to calculate the one-step-ahead CoVaR prediction:

(a) To estimate the model parameters, we use the *Inference Function for Margins* (IFM) method proposed by Joe and Xu (1996). This is a two-step maximum likelihood procedure for parametric copulas. First, we estimate the marginal models. Thus, for each of the samples $r_{1,1},\ldots\ldots,r_{1,T}$ and $r_{2,1},\ldots\ldots,r_{2,T}$ we fit the univariate model (6) - (9) to obtain estimates of $\mu_{i,t}$ and $\sigma_{i,t}$ denoted by $\hat{\mu}_{i,t}$ and $\hat{\sigma}_{i,t}$ respectively. We calculate $\hat{\mu}_{i,t} = \hat{F}_i(\hat{\eta}_{i,t})$ for $i = 1,2$, $t = 1,\ldots\ldots, T$, where $\hat{F}$ is the empirical distribution function[3] and $\hat{\eta}_{i,t} = (r_{i,t} - \hat{\mu}_{i,t})/\hat{\sigma}_{i,t}$ is the estimated innovation. Second, the copula parameters (q) are estimated by maximizing the expression

$$L(\theta) = \sum_{t=1}^{T} \log c\left(\hat{\mu}_{1,t}\ \hat{\mu}_{2,t};\theta\right) \tag{16}$$

for a specific copula density c. For example, when using the *t*-Student copula (14): $\theta = (\nu,\rho)$ and when using the Joe-Clayton copula, (15): $\theta = (\tau_L,\tau_U)$.

[3] This non-parametric estimator is usually employed in copula applications. However, we might also use the distribution function of $\eta_{i,t}$, which is GED.

(b) To generate the observations:

(b.1) Simulate m copula samples $U=[u_{1,j},u_{2,j}]$, j = 1,………,m, using the estimated copula parameters ($\hat{\theta}$).

(b.2) Calculate $Z_{i,j}=F_i^{-1}(u_{i,j})$ for i = 1,2 and j = 1,………,m, where $F_i^{-1}(.)$ is the inverse GED distribution with parameter $\hat{\nu}_i$.

(b.3) Using the one-step-ahead predictions of the conditional means and conditional variances, denoted by $\hat{\mu}_{i,t+1}$ and $\hat{\sigma}_{i,t+1}$, respectively, calculate $r_{i,t+1,j}=\hat{\mu}_{i,t+1}+\hat{\sigma}_{i,t+1}\,Z_{i,j}$ for i = 1, 2 and j = 1,………,m.

(b.4) Denote the simulated observations $(r_{1,t+1,j},r_{2,t+1,j})$ as (x_j^*,y_j^*) for j = 1,..,m. Thus, the (x, y) bivariate sample of interest is $(x_1^*,y_1^*),\ldots,(x_m^*,y_m^*)$.

(c) Finally, calculate the VaR and CoVaR using empirical quantiles. The $VaR^x(p)$ is the p-quantile of the simulated observations $x_1^*,\ldots..,x_m^*$ and the $CoVaR^{(y|x)}(q,p)$ is the q-quantile of the y_j^* observations from the pairs (x_j^*,y_j^*) which the x_j^*'s are lower or equal than $VaR^x(p)$. In addition, the unconditional VaR of the y series, denoted by $VaR^x(q)$ is estimated as the q-quantile of the simulated observations $y_1^*,\ldots.,y_m^*$.

We emphasize that in step (b.3) we use the predicted volatility: $\hat{\sigma}_{i,t+1}$. This is because we want to consider a real scenario in which the risk measures have to be estimated from the available information.

On the other hand, to calculate the CoVaR variation we follow the procedure described above, the only difference being that in (b.3) the CoVaR during financial distress and normal periods are calculated using conditional means and conditional variances estimates: $\hat{\mu}_t$ and $\hat{\sigma}_t$, respectively, not predictions. Additionally, the CoVaR in normal periods is calculated as the q-quantile of the y_j^* from pairs (x_j^*,y_j^*) in which the x_j^* are lower than or equal to the Q_3 quantile of x^* and bigger than the Q_1 quantile of x^*.

3. EMPIRICAL ANALYSIS

In this section we present the application of the proposed method in estimating the CoVaR of the Peruvian stock market. All calculations were performed using the statistical software package R (R Development Core Team, 2014). We begin by presenting the data.

3.1. The Data

The data used in this paper are the daily closing prices of the IGBVL, the S&P500 index of the USA, and the prices of copper, gold and silver traded on the London Metal Exchange (LME). The data source is Bloomberg. The sample covers the period starting on January 2, 2004 and ending on December 31, 2013. Because the three markets have different holidays from one another, we only consider the days in which all prices are available.

We chose the IGBVL as representative of the Peruvian stock market, and the S&P500 index as representative of the international stock market. In addition, we consider the international prices of copper, gold and silver for two reasons: because the stocks of Peruvian mining companies are very important in the composition of the IGBVL, and because these metals are three of the main commodities exported by Peru (Peru's export earnings economy depends critically on commodities).

In this paper we study daily time-series returns, in percentages, of the IGBVL, S&P500, copper and gold-silver. The latter corresponds to the daily average of gold and silver returns. We consider this basket because the historical prices of both precious metals are similar during the period, see Zevallos and del Carpio (2015). Specifically, returns were calculated as $r_t = 100(\ln(p_t) - \ln(p_{t-1}))$, where p_t is the price at time t and ln is the natural logarithm.

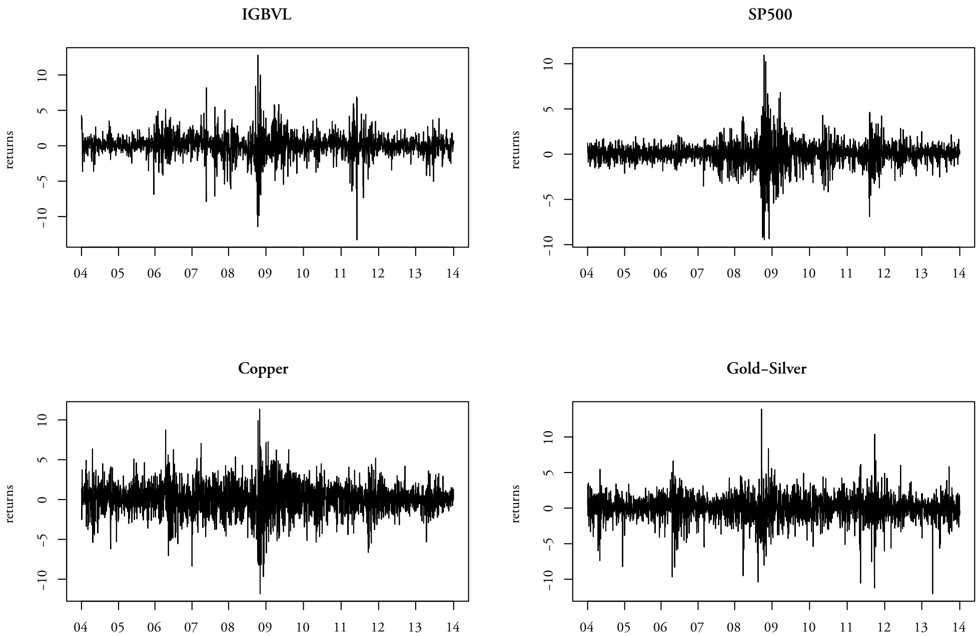
Each time series consists of T = 2372 returns. Table 1 present some descriptive statistics and Figure 1 shows the graph of the time series returns. In this figure we can clearly identify periods of high volatility. For example, the four series exhibit very high volatility in the second half of 2008, and the IGBVL series present episodes of high volatility during Peruvian presidential election periods: in 2006 and 2011.

Table 1. Descriptive statistics of daily returns in the period 2004-2013.

Series	T	Mean	Standard Deviation	Skewness	Kurtosis
IGBVL	2372	0.078	1.702	-0.43	11.07
S&P500	2372	0.022	1.305	-0.38	13.90
copper	2372	0.048	2.002	-0.17	5.49
gold-silver	2372	0.047	1.846	-0.53	8.66

[a] T is the number of observations.

Figure 1: Return series.



3.2. Estimation of the Stock Market Risk

Peruvian stock market risk, measured through the risk of the IGBVL, was analyzed in three different periods according to three different IGBVL volatility levels. For instance, we consider 2013 a period of low and medium volatility; and both July-December 2008 (which includes the Lehman Brothers bankruptcy), and January-August 2011 (which includes the presidential election) as periods of high volatility,

In each of these three periods we estimate the one-step-ahead CoVaR predictions of the IGBVL, conditional on S&P500, conditional on copper and conditional on gold-silver. We also estimate the ΔCoVaR of the IGBVL conditional on S&P500, conditional on copper and conditional on gold-silver. Regarding the choice of confidence levels, we adopt a conservative approach in terms of CoVaR estimation, using $q = 0.01$ and for conditioning on the VaR we considered a less extreme value, $p = 0.05$. In all subsequent discussions we will omit $p = 0.05$ and simply refer to CoVaR 99% and ΔCoVaR 99%[4]. Additionally, we calculate unconditional one-step-ahead VaR 99% predictions for the IGBVL.

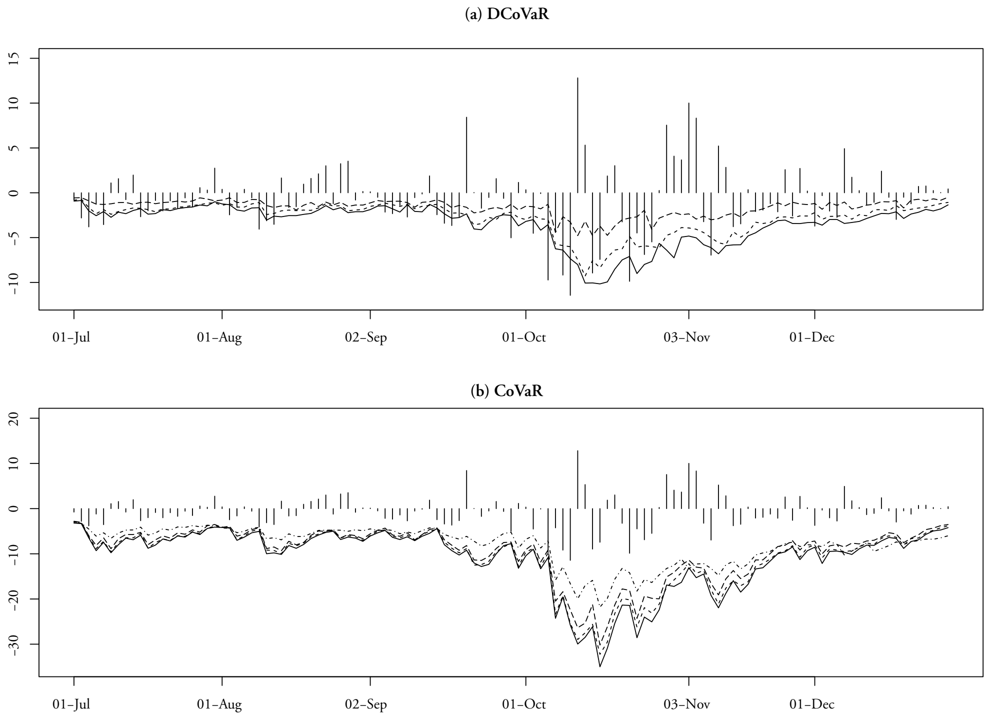
At each time, we estimate the risk measures following the procedure described in Subsection 2.2. Specifically, we estimate models with first order autoregressive component for the IGBVL and S&P500, and models without autoregressive component for copper and gold-silver. We consider APARCH(1,1) models with leverage and errors GED;

[4] Following the convention, confidence of 99% refers to the losses, *i*.e. negative values of returns.

in this way we can capture the effect of positive and negative returns (leverage) and heavy tails. The copulas we consider are Joe-Clayton and *t*-Student, and we choose the one that best explains the bivariate dependence. At each time m = 100,000 observations were simulated. We emphasize that the CoVaR and ΔCoVaR are calculated each time a new observation is available in the sample. Let $t = 1, \ldots.. t_{0-1}, t_0, \ldots.., T$ the times in the period. The returns at times $t = 1, \ldots, t_{0-1}$ constitute the basic sample, and then we include the returns in the sample one by one to calculate CoVaR at $t = t_0, \ldots, T$. Thus, for the period July to December 2008 we have T = 1175 and t_0 = 1057, for the period January-August 2011 we have T = 1812 and t_0 = 1674, and for the period January to December 2013 we have T = 2372 and t_0 = 2130.

The results are shown in figures 2-4. Figures 2a, 3a and 4a show the CoVaR variations in the three considered periods, and figures 2b, 3b and 4b show the one-step-ahead VaR[5] and CoVaR predictions for the three considered periods. From them we can state the following.

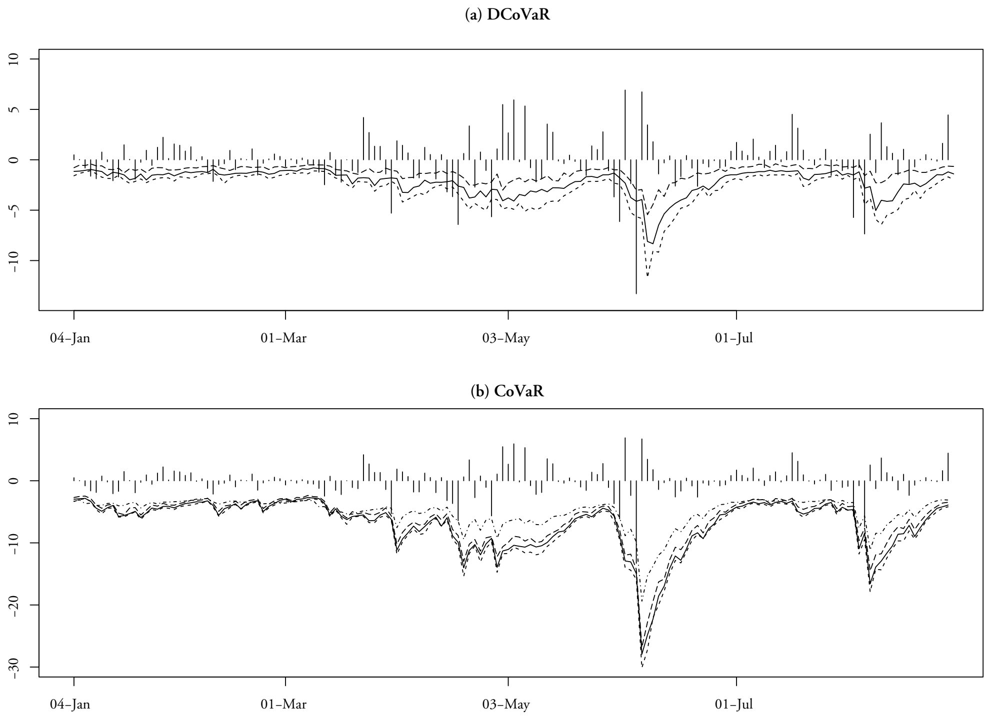
Figure 2. July-December 2008 period.



(a) IGBVL 99% ΔCoVaR evolution. IGBVL returns in vertical lines, ΔCoVaR conditional on S&P500 in dashed lines (---), ΔCoVaR conditional on copper in solid lines (—), and ΔCoVaR conditional on gold-silver in longdash lines (— —). (b) IGBVL risk. IGBVL returns on vertical lines, IGBVL 99% VaR in two dashed lines (•-•-), CoVaR conditional on S&P500 in dashed lines (---), CoVaR conditional on copper in solid lines (—), and CoVaR conditional on gold-silver in longdash lines (— —).

5 Corresponding to the bivariate series: IGBVL and copper.

Figure 3. January-August 2011 period.



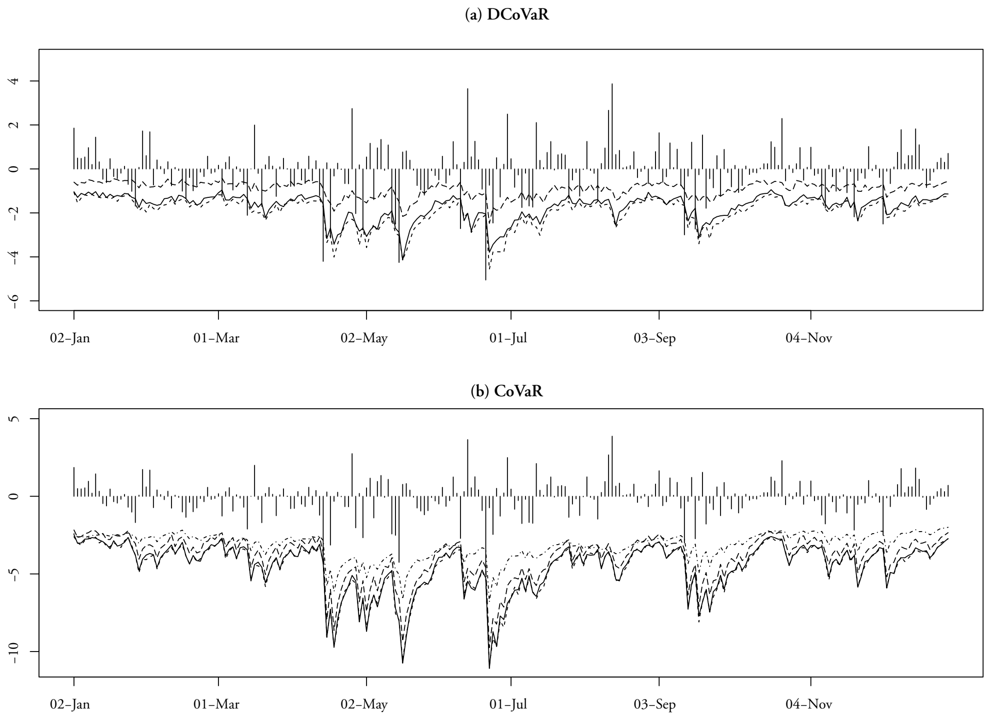
(a) IGBVL 99% ΔCoVaR evolution. IGBVL returns in vertical lines, ΔCoVaR conditional on S&P500 in dashed lines (---), ΔCoVaR conditional on copper in solid lines (—), and ΔCoVaR conditional on gold-silver in longdash lines (— —). (b) IGBVL risk. IGBVL returns on vertical lines, IGBVL 99% VaR in two dashed lines (•-•-), CoVaR conditional on S&P500 in dashed lines (---), CoVaR conditional on copper in solid lines (—), and CoVaR conditional on gold-silver in longdash lines (— —).

First, both the estimates of ΔCoVaR and one-step-ahead CoVaR predictions exhibit time-varying behavior, in line with the IGBVL volatility. However, the relationship between the VaR of IGBVL and the CoVaR is not one-to-one. This feature was also found by Adrian and Brunnermeier (2011).

Second, we will discuss the marginal contribution of S&P500, copper and gold-silver to the IGBVL's risk, that is, the CoVaR variations (ΔCoVaR). In figures 2a and 3a we note that in periods of low IGBVL volatility, the ΔCoVaR given S&P500 and ΔCoVaR copper are very close, with the ΔCoVaR given S&P500 slightly higher (the same occurs in a period of low and medium IGBVL volatility: 2013, see Figure 4a). However, the differences between the ΔCoVaR given S&P500 and ΔCoVaR given copper become greater when the IGBVL volatility increases. This is clearly seen in the case of October of 2008[6] (Figure 2a) and during the 2011 election period (Figure 3a).

[6] In this period, the USA began the implementation of the Troubled Asset Relief Program (TARP), a program designed to save the financial system.

[Figure 4]



(a) IGBVL 99% ΔCoVaR evolution. IGBVL returns in vertical lines, ΔCoVaR conditional on S&P500 in dashed lines (---), ΔCoVaR conditional on copper in solid lines (—), and ΔCoVaR conditional on gold-silver in longdash lines (— —). (b) IGBVL risk. IGBVL returns on vertical lines, IGBVL 99% VaR in two dashed lines (•-•-), CoVaR conditional on S&P500 in dashed lines (---), CoVaR conditional on copper in solid lines (—), and CoVaR conditional on gold-silver in longdash lines (— —).

In addition, during the period from July to December of 2008, the marginal contribution to risk due to copper is bigger in magnitude than the marginal contribution due to S&P500; while during January-August of 2011, this relationship is reversed: in the three periods the ΔCoVaR conditional on gold-silver is smaller in magnitude than the ΔCoVaR conditional on S&P500 and conditional on copper. Therefore, the marginal contribution of gold-silver to IGBVL's risk is low in the three considered periods and very low in periods of high IGBVL volatility. This relates to the usual practice of using gold and silver as a hedge in portfolio construction.

Third, we will discuss one-step-ahead IGBVL risk predictions. For this, in each period we compare IGBVL VaR against CoVaR conditional on S&P500, copper and gold-silver. We start by analyzing figures 2b, 3b and 4b.

With the exception of very few days (and during periods of IGBVL low volatility), IGBVL CoVaR predictions are lower than VaR predictions. This is desirable, given the definition of CoVaR and because the objective is to have a more conservative risk measure[7].

During the three periods considered: July-December 2008, January-August 2011 and 2013, CoVaR predictions are close to one another and CoVaR conditional on gold-silver is usually lower than CoVaR conditional on copper and S&P500. Similarly, during periods of low and medium volatility, IGBVL VaR predictions and CoVaR predictions are close to one another, but differences between VaR and CoVaR become larger during periods of high volatility. This observed behavior can be interpreted as a non-linear risk response captured by using CoVaR during financial distress periods.

Like VaR, CoVaR is very sensitive to big sequential losses (large negative returns). For example, in Figure 2b we analyze the period corresponding to the second half of 2008, which includes the Lehman-Brother bankruptcy. At the beginning of the series, we observe that the VaR and the two CoVaR estimates have close values; however, at the peak of the crisis the values for VaR and CoVaR estimates are very large, reaching around 20% for VaR and 30% for CoVaR correspondingly. Once after the peak of the crisis, the measurements of risk become close again by the end of the time series. A similar situation can be observed in Figure 3b, where there is a large increase in risk measurements around the 2011 presidential elections.

To compare the coverage of each risk measurement, in Table 2 we show the number of exceptions, that is, the number of times when actual returns where lower than VaR and CoVaR predictions at 99% for each of the three time periods considered. The expected number of exceptions are 1.19, 1.58 and 2.43 for July-December 2008, January-August 2011 and January-December 2013, respectively. The results show that unconditional VaR does not adequately cover losses for the January-August 2011 and January-December 2013 time periods. By contrast, each CoVaR has fewer exceptions than expected, which makes CoVaR better at adequately covering losses than unconditional VaR.

Table 2. Number of returns lower than risk measures at 99%.

Period	*n*	VaR	CoVaR-S&P500	CoVaR-copper	CoVaR-gold-silver
July-December 2008	119	1	0	0	1
January-August 2011	158	5	1	1	1
January-December 2013	243	4	0	0	2

[a] *n* is the number of observations in each period.

However, when assessing a risk measure, besides analyzing coverage it is also important to analyze the absolute size of the values of this measure. In a time of medium volatility with some high volatility spikes, like 2013 (Figure 4b), CoVaR estimates

7 Depending on the estimation method, CoVaR values can often be lower than VaR. This happened, for example, when we estimated the CoVaR using quantile regression and VaR using GARCH models.

adequately cover the observed losses. However, as can be seen in figures 2b and 3b, during time periods of intense volatility both VaR and CoVar values could be too large, in the range of 20% - 30%. We can explain the presence of unnecessarily large CoVaR estimates compared to the observed returns over the course of several days through the following example. Let us suppose we have an atypical (negative) return value, followed over time by several small returns. Under the volatility model adopted in Equation (9), the day after the atypical value event, volatility (and therefore, VaR and CoVaR) become considerably larger and then slowly decrease during the days after, still with small returns, thus showing that the volatility model is not adaptive enough for this situation.

Finally, in Table 3 we present IGBVL returns and the risk measures from *exception days* found during the three periods. The biggest loss happened on June 6, 2011, the day after the results of the second round of the 2011 presidential election were announced. Note that CoVaR predictions are able to cover those day losses, while VaR does not. However, on August 4, 2011 none of the risk measures were able to adequately cover the loss. The difference in this behavior is explained by the IGBVL volatility level in the days before the prediction. The days preceding June 6, 2011 had already witnessed high volatility, with returns of -3.68 (May 31), -6.13 (June 1), 6.92 (June 2), and -2.37 (June 3). On the other hand, the days before August 8 had witnessed low volatility, with returns of -0.09 (August 1), -1.017 (August 2), and -0.89 (August 3). Therefore, the observed return on August 4 of -5.73 is very hard to predict given the low volatility levels previously observed.

Table 3. Risk measures at 99% in the *exception days*.

Date	Return IGBVL	VaR	CoVaR S&P500	CoVaR copper	CoVaR gold-silver
06/10/2008	-9.73	-7.13	-9.77	-10.81	-9.13
28/03/2011	-5.29	-4.42	-6.39	-6.23	-5.66
01/06/2011	-6.13	-5.95	-9.47	-8.90	-7.92
06/06/2011	-13.29	-9.87	-15.87	-14.94	-14.27
04/08/2011	-5.73	-3.49	-4.82	-4.64	-4.08
08/08/2011	-7.36	-5.63	-9.46	-8.43	-7.64
15/04/2013	-4.20	-2.86	-4.29	-4.31	-3.65
20/06/2013	-5.05	-3.40	-5.31	-5.27	-4.59
12/09/2013	-3.00	-2.73	-4.10	-3.83	-3.43
03/12/2013	-2.50	-2.26	-3.44	-3.38	-3.06

4. CONCLUSIONS AND FURTHER RESEARCH

In this study, we estimate the Peruvian stock market risk considering two sources of risk: international financial market risk, represented by S&P500; and the risk from the three main Peruvian export products of copper, silver and gold. For this purpose, we propose a methodology based on the concepts of CoVaR and delta CoVaR from Adrian and Brunnermeier (2008)

Our results illustrate the usefulness of the CoVaR methodology. In particular, we discuss two aspects: the use of CoVaR variation and the use of CoVaR prediction.

Using CoVaR variation, we find that the contribution to IGBVL risk from a basket of gold-silver is lower than the contribution of S&P500 and copper, with contributions from S&P500, copper, and gold-silver being very close during normal market time periods.

In terms of prediction, the results show the usefulness of CoVaR measures of risk conditional on S&P500, conditional on copper, and conditional on gold-silver as alternatives to a measure such as unconditional VaR. In this sense, one-step-ahead CoVaR predictions provide better coverage in terms of number of exceptions than VaR, and can adequately cover losses during periods of low and medium-high volatility. Even further, unlike VaR, CoVaR is able to cover extreme losses like those of June 6, 2011, the day after the second round of the Peruvian elections. However, during periods of very high volatility and under the situations described in the previous section, the values of VaR and CoVaR could be unnecessarily large. In these cases, the adoption of a 10% margin from the Peruvian bank regulator, as indicated by SBS Resolution No 6328, becomes relevant.

Even if CoVaR estimates under high-volatility episodes are too large, the calculation of CoVaR shows a bigger picture incorporating exogenous risk. All of this is in line with the recent global financial crisis, started in 2008, which emphasized the importance of capturing risk transmission and highlighted the disadvantages of adopting low capital coverage levels.

Besides, crisis periods trigger non-linear market risk responses, as evidenced by the increment of differences between VaR and CoVaR during periods of high volatility compared to normal volatility. The same was found with CoVaR variation.

Finally, we can identify two future topics of study. First, to evaluate CoVaR methodology on stock portfolio risk estimation by measuring the systemic risk of portfolios against IGBVL and other portfolios, and vice versa. Second, to propose alternative CoVaR estimation methods to improve adaptability to the occurrence of small losses after extreme ones.

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