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Environmental Policy, Mergers and Environmental R&D with Spillovers

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Abstract

This project lies at the frontier between environmental economics and industrial organization. We use a duopoly setting of a three-stage game; in the first stage, the government chooses an emission tax and aims for maximizing welfare; in the second stage, firms use R&D to reduce their emissions; in the last stage, firms compete a la Cournot with differentiated products. We focus on two policy regimes and three scenarios, namely regimes of competition and merger and scenarios of commitment, non-commitment, and exogenous tax. The study focuses on two major questions: (1) what is the effect of merger on R&D, and the effect of commitment on R&D? (2) what is the effect of merger and commitment on the economy? Results are obtained through numerical simulations of the model. We find that: (i) Merger has a positive effect on R&D under non-commitment and the exogenous tax scenarios. (ii) Under commitment, if goods are imperfect substitutes or homogenous, merger has a negative effect on R&D; if goods are complements or independent, merger has a positive effect on R&D. (iii) For any types of goods under any regime, commitment has a negative effect on R&D.

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Declaration of interests

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

1. Introduction

Pollution problems such as carbon emissions have become a major issue faced by modern governments. In order to solve this problem, we need to have a better understanding about the underlying mechanism that leads to carbon pollution. This includes understanding the economic theories associated with this type of market failure, which are based on environmental economics and industrial organization. Comprehensive study in those two fields would help us to design effective policies to provide both emission-reducing incentives for major polluting industries as well as increases in overall welfare.

The related literature focuses on several different aspects of the issue. Atallah (2016) discusses the relationship between competition and innovation. Poyago-Theotoky (1999) analyzes a duopoly where firms are engaged in cost reducing R&D and then compete in quantities. Poyago-Theotoky (2007) concentrates on an industry consisting of two firms selling a homogenous product. Petrakis and Poyago-Theotoky (2002) study the impact of two technology policies of R&D subsidization and R&D cooperation, in a duopoly model with cost-reducing R&D, spillovers and pollution abatement R&D. Additionally, Ouchida and Goto (2011) summarize Poyago-Theotoky (2007): for small environmental damages and inefficient R&D technologies, R&D and welfare are both higher in the R&D cartel than under non-cooperative R&D. They also prove that Poyago-Theotoky's (2007; 2010) results are still robust in a relaxed wider parameter range of environmental damage coefficient. Ouchida and Goto (2016a) is based on Poyago-Theotoky and Teerasuwannajak (2002), and it extends this previous work by considering that the government has a pre-commitment ability to an emission tax into the model of Cournot duopolists with end-of-pipe technology. Ouchida and Goto (2016b) first assume that the regulator has no pre-commitment ability to an emission tax, then they present an examination of four types of R&D in a Cournot duopoly: environmental research joint venture cartelization, R&D competition, R&D cartelization, and environmental research joint venture competition.

We use a duopoly setting of a three-stage game; in the first stage, the government chooses an emission tax and aims for maximizing welfare; in the second stage, firms use R&D to reduce their emissions; in the last stage, firms compete a la Cournot with differentiated products. We focus on two policy regimes and three scenarios, namely regimes of competition and merger and scenarios of commitment, non-commitment, and exogenous tax. Commitment refers to the regulator who chooses a tax in the first stage, before the firm chooses R&D, and then the regulator commits to this tax. Non-commitment is when a firm chooses R&D in the first stage, then the regulator chooses a tax in the second stage. Past industrial merger experiences show that polluting firms have a significant share in the total value and volume of mergers, therefore it is important to study the mergers in the context of environmental regulation. In addition, simulation model suggests that the results depend on whether the government has the commitment ability to an emission tax or not, therefore it is interesting to analyze the commitment vs non-commitment scenarios. We also take into account the exogenous tax scenario since it is relevant when the tax is set based on country-wide considerations or on an international agreement, and hence is not responsive to the specific conditions of the industry under study.

The study focuses on two major questions: (1) what is the effect of merger on R&D, and the effect of commitment on R&D? (2) what is the effect of merger and commitment on the economy? Results are obtained through numerical simulations of the model. We find that: (i) Merger has a positive effect on R&D under non-commitment and the exogenous tax scenarios. (ii) Under commitment, if goods are imperfect substitutes or homogenous, merger has a negative effect on R&D; if goods are complements or independent, merger has a positive effect on R&D. (iii) For any types of goods under any regime, commitment has a negative effect on R&D.

Our paper makes the following contributions to the literature: first, our research focuses on total merger, which is cooperation at all stages. Previously, most studies have only focused on the regime of R&D cooperation. Second, our study fully considers product differentiation. This is different from major strands of the literature which only study a homogenous product. In contrast, our research considers four types of goods: complements, independent, imperfect substitutes, and homogenous. Third, our research includes three different scenarios: commitment, non-commitment, and exogenous tax. Other major studies only focus on one or two of them (e.g. Poyago-Theotoky, 1999, 2007, 2010; Petrakis and Poyago-Theotoky, 2002; Ouchida and Goto, 2011, 2014, 2016a,b).

This paper is organised as follows. The following section reviews the literature and Section 3 introduces the model. The fourth section is an exploration of the model under commitment, while the fifth section is an exploration of the model under non-commitment, and the sixth section presents an examination of the model under the exogenous tax scenario. The seventh section examines Cournot duopolists' equilibrium variables between commitment and non-commitment within each policy regime. The eighth section presents the major results of the relationship between merger and R&D and the relationship between commitment and R&D. The last section presents the conclusions.

2. Literature Review

Atallah (2016) discusses the relationship between competition and innovation. This relationship can be analyzed through either the effect of competition on innovation outputs in terms of effective cost reduction or the effect of competition on innovation inputs in terms of R&D expenditure. For example, Vives (2008) examines this relationship through competition and innovation inputs. We also notice that competition can take two forms in his model: an increase in the number of firms as from merger to competition, and a decrease in product differentiation. Atallah's (2016) results show that, for a wide range of parameters, the relationship between competition and innovation is more often negative than positive.

Poyago-Theotoky (1999) analyzes a duopoly where firms invest in R&D and then compete in quantities, in a three-stage game of almost perfect information. In the first stage, two identical firms simultaneously choose their R&D expenditure. In the second stage, the two rivals decide how much of the knowledge created in the first stage to disclose. In the third stage, firms play a standard Cournot game, given the cost levels and spillovers achieved in the previous stages. Results show that when choosing R&D non-cooperatively, firms never disclose any of their information, whereas when choosing R&D cooperatively, firms always fully share their information. As a result, public policies should focus on encouraging pre-competitive R&D cooperation, since R&D cooperation not only leads firms to engage in more R&D in terms of increasing innovation output, but also has the beneficial effect of making firms fully disclose their information which leads to less duplication and more coordination of research activities. Firm profitability and welfare will increase correspondingly. Poyago-Theotoky (1999) shows that R&D cooperation leads to higher innovation output in terms of effective cost reduction compared with R&D competition.

Poyago-Theotoky (2007) concentrates on an industry consisting of two firms selling a homogenous product. One firm undertakes R&D to reduce pollution and receives an abatement benefit from its rival through positive spillovers. Firms choose their emission-reducing R&D non-cooperatively or cooperatively, then the regulator sets the emission tax, and firms compete in quantities. She concludes that for small damages, R&D and welfare are higher in the R&D cartel; for large damages and efficient R&D, R&D and welfare are higher under R&D competition. Poyago-Theotoky (2007) asserts that there are three policy instruments associated with three types of market failures: a pollution tax to address the environmental externality, an R&D subsidy to address the R&D market failure, and a research joint venture to correct the information-sharing market failure. Poyago-Theotoky (2007) also believes competition in an innovation market will result in two market failures: (1) if spillovers are absent, competition leads to a strategic over-investment effect; and (2) under positive spillovers, competition leads to a strategic under-investment effect. Poyago-Theotoky (2007) defines innovation output as R&D contributions towards emission reduction of harmful pollutants, and concludes that a three-stage game of quantity competition will generate a higher innovation output in R&D cartel relative to R&D competition i) if the environmental damage is low as well as ii) if there is a large damage parameter and a large R&D cost parameter.

Poyago-Theotoky (2010) shows that if there is a small damage parameter and R&D is relatively efficient, emission subsidies can partially correct for the inefficiency caused by firms' market power. Under this circumstance, the environmental externality is offset by the under-production market failure.

Poyago-Theotoky and Teerasuwannajak (2002) consider that product differentiation can play a role in the environmental policy design and implementation under both the regulator's full commitment and no commitment. In their model, they do not have spillovers and they assume production cost is zero. Their conclusions are: 1) if products are highly differentiated, the optimal no-commitment tax is always lower than the optimal pre-commitment tax. As products become more and more similar, for both a low level of efficiency parameter and a high level of damage parameter, the optimal no-commitment tax is greater than the optimal pre-commitment tax; and (2) if product differentiation is high, abatement and welfare will be higher under the no-commitment policy regime unless the efficiency parameter is extremely low. As products become less differentiated, abatement and welfare will be higher under the pre-commitment regime. Their paper uses pollution reduction as the innovation output, and by comparing competition between two policies for a given degree of product differentiation, the efficient abatement technology implies more innovation under commitment. However, as the degree of product differentiation decreases, more intensive competition leads to more innovation under non-commitment.

Petrakis and Poyago-Theotoky (2002) study the impact of two technology policies of R&D subsidization and R&D cooperation in a duopoly model with cost-reducing R&D, spillovers and pollution abatement R&D. They model two policy instruments as two different regimes and aim to compare welfare between the two technology policies. They point out that even though cost-reducing R&D could lead to firms generating less pollution per unit of output produced, there can nevertheless be a case where total pollution will still increase due to increased production. They use an exogenous tax combined with an R&D subsidy and a research joint venue to address three types of market failure: environmental externality, innovation market failure and information sharing market failure. They show two important results which are different from the conventional literature pertaining to the positive R&D subsidies and desirability of R&D cooperation: (1) if the carbon tax is exogenous and firms have cost-reducing R&D and abatement innovation, the optimal R&D subsidy can be negative. Too much cost-reducing R&D leads to more production; a negative R&D subsidy is optimal when the damage parameter is large and the exogenous tax is low; and (2) welfare comparison between the two policy regimes hinges on the spillover rate, damage parameter and tax. In most cases, welfare is higher under R&D subsidization than under R&D cooperation.

In their 2011 study, Ouchida and Goto summarize Poyago-Theotoky (2007): for small environmental damages and inefficient R&D technologies, R&D and welfare are both higher in the R&D cartel than under non-cooperative R&D. They also prove that Poyago-Theotoky's (2007; 2010) work is still robust in a relaxed wider parameter range of environmental damage coefficient. They also conclude that as the degree of product differentiation increases, the damage parameter set which guarantees an interior solution of R&D will increase as well. Their paper is different from our model in two aspects: first is the organization of R&D, where we have total merger instead of R&D cooperation; second, in Ouchida and Goto (2011), the regulator is unable to commit to an emission tax, whereas in our paper we have both scenarios of commitment and non-commitment.

Ouchida and Goto (2014) analyze the circumstances in which a negative emission tax occurs and reduces emissions. Several findings are illustrated, including: (1) an emission subsidy under R&D cartelization increases welfare compared with *laissez-faire*, (2) a very small environmental damage parameter implies a negative equilibrium emission tax. The economic intuition is as follows: an emission subsidy is equivalent to the policy combination of a production subsidy and an abatement tax, since pollution is generated by the firm's production minus net abatements. A production subsidy consists of two effects of damage-increasing effect and decreasing effect of market inefficiency. When damage is sufficiently small, the damage-increasing effect is dominated by the improvement in market inefficiency; and (3) if the damage parameter is sufficiently small and R&D cost is low, total emissions under a non-commitment emission subsidy policy are lower than under *laissez-faire*, i.e., the emission subsidy has an emission-reducing effect. The economic intuition underlying the emission-reducing effect is that the emission-increasing effect is dominated by the large abatement effect.

The study of Ouchida and Goto (2016a) is based on Poyago-Theotoky and Teerasuwannajak (2002). It expands on this previous work by considering that the government has a pre-commitment ability to an emissions tax into the model of Cournot duopolists with end-of-pipe technology. Then, they study the model's effects on welfare and other equilibrium values of variables under R&D cartelization and R&D competition. The following conclusions are made: (1) if the spillover parameter is positive, the regulator always prefers R&D cartelization to R&D competition; (2) although environmental research joint venture cartelization creates certain social and private incentives for firms, it cannot guarantee maximum consumer surplus; (3) a large damage parameter and a small R&D cost parameter guarantee that environmental research joint venture cartelization is socially efficient from both a consumer surplus and welfare point of view; and (4) under positive spillovers, Cournot duopolists competition with R&D cartelization generates more innovation outputs in terms of abatements.

Ouchida and Goto (2016b) assume that a regulator has no pre-commitment ability to an emission tax, then examine four types of R&D in a Cournot duopoly: environmental research joint venture cartelization, R&D competition, R&D cartelization, and environmental research joint venture competition. After that, they present a comprehensive evaluation of the welfare performance of the four R&D formations and examine the government's competition policy, concluding that: (1) a large damage parameter and a low R&D cost parameter imply that R&D competition is better than other scenarios in terms of welfare; (2) with a high damage parameter and a high R&D cost parameter, environmental research joint venture cartelization is the optimal policy; (3) environmental research joint venture cartelization yields higher profits; and (4) despite the fact that Japanese antitrust authorities give a slight degree of tolerance to environmental research joint venture cartelization, their paper nevertheless shows that environmental research joint venture cartelization generates welfare benefits and therefore these policy implications should apply to current antitrust policy systems.

Our paper makes the following contributions to the literature: first, our research focuses on total merger, which is cooperation at all stages. Previously, most studies have only focused on R&D cooperation. Second, our study fully considers product differentiation. This is different from major strands of the literature which only study a homogenous product. In contrast, our research considers four types of goods: complements, independent, imperfect substitutes, and homogenous. Third, our research includes three different scenarios: commitment, non-commitment, and exogenous tax. Other major studies only focus on one or two of them (e.g. Poyago-Theotoky, 1999, 2007, 2010; Petrakis and Poyago-Theotoky, 2002; Ouchida and Goto, 2011, 2014, 2016a,b).

3. The Model

The model used is a duopoly model with product differentiation. Each firm produces one differentiated product and generates pollution. The government uses an emission tax t as an environmental policy instrument. Each firm makes an R&D investment for pollution reduction[1] and R&D spillovers exist between firms. R&D takes the form of pre-competitive R&D: pre-competitive R&D is done before product market competition. There are three stages in this game: the government chooses the emission tax, firms choose R&D, and firms compete in output. There are two policy regimes and three scenarios. The first regime is competition at all stages, where both firms act non-cooperatively and each firm chooses R&D and output to maximize its profit. The second regime is merger, where all decisions maximize joint profit. After the merger, the new entity continues to produce two products. In each regime, there are three scenarios: (1) the regulator is able to commit to an emission tax, (2) the regulator is unable to commit to an emission tax, and (3) the emission tax is exogenous. Commitment refers to the regulator who chooses a tax in the first stage, before the firm chooses R&D, and then the regulator commits to this tax. Non-commitment is when a firm chooses R&D in the first stage, then the regulator chooses a tax in the second stage. Our goal is (1) to study the effect of merger on innovation and the effect of commitment on innovation; and (2) to study the effect of merger and of commitment on the economy in terms of R&D spending, pollution reduction, pollution, welfare and other variables.

Specifically: Two firms (i and j) are engaged in quantity competition. The firms are selling differentiated goods. Each firm faces a linear demand as

$$p_i(q_i, q_j) = a - (q_i + \theta q_j), (i, j = 1, 2; i \neq j), \tag{1}$$

where $a > 0$ is a market size parameter. Product differentiation is captured by $\theta \in (-1, 1]$. If $\theta \in (-1, 0)$ products are complements; if $\theta = 0$ products are independent; if $\theta \in (0, 1)$ products are imperfect substitutes; if $\theta = 1$ products are homogenous.

Both firms have the same cost structure of

$$C(q_i) = cq_i. \tag{2}$$

Both firms use end-of-pipe technologies as their emissions-reducing technology.[2] End-of-pipe technologies abate emissions by adsorbing emissions at the end of the production process.

R&D and the cost structure include the following features: the value of each firm's emissions per unit of output is assumed to be one. Firm i's R&D effort is denoted as z_i. R&D effort means firm i can abate its emissions as z_i units. Firm i receives benefits both from its own R&D efforts and from efforts of its rival. Poyago-Theotoky (1999, 2007, 2010) and Ouchida and Goto (2011, 2014, 2016a,b) assume the R&D expenditure function is quadratic, our research follows this path. We assume the R&D expenditures for firm i is increasing in z_i. For z_i units of abatement the

[1] The benchmark case of pollution reduction is no tax and no R&D. If there is a tax, there is pollution reduction.

[2] Petroleum and chemical industries typically have end-of-pipe technologies for pollution abatement such as desulfurization equipment and denitrification equipment.

associated R&D expenditures for firm i are defined as

$$\left(\frac{\gamma}{2}\right)z_i^2. \tag{3}$$

A lower value of γ implies a higher efficiency of R&D, where $\gamma > 0$.

The spillover effect of R&D is $\beta \in [0,1]$. Firm i's positive externality from the rival's R&D efforts is denoted as βz_j. The emission function is

$$e_i(q_i, z_i) = q_i - z_i - \beta z_j. \tag{4}$$

Firm i can abate its emissions from q_i to e_i.

The representative consumer's utility function is

$$U(q_i, q_j) = a(q_i + q_j) - \left(\frac{1}{2}\right)(q_i^2 + 2\theta q_i q_j + q_j^2) + m, \tag{5}$$

where m denotes the consumption of a numeraire good. Consumer's utility function does not include pollution effects.

As derived in the Appendix, consumer surplus is given by

$$CS = \left(\frac{1}{2}\right)(q_i^2 + 2\theta q_i q_j + q_j^2). \tag{6}$$

Total producers' profit is given by

$$\text{Profit} = \pi_i + \pi_j, \tag{7}$$

where π_i is defined as

$$\pi_i = p_i(q_i, q_j)q_i - cq_i - t(q_i - z_i - \beta z_j) - \left(\frac{\gamma}{2}\right)z_i^2,\ (i, j = 1, 2; i \neq j). \tag{8}$$

Emissions cause environmental damage. The environmental damage is

$$D(E) \ = \left(\frac{d}{2}\right)E^2, \tag{9}$$

where E is defined as

$$E = e_i + e_j, \tag{10}$$

and d represents the damage parameter. Government tax revenue is equal to

$$\text{Tax Revenue} = tE. \tag{11}$$

Welfare is defined as

$$W = CS + \text{Profit} - \text{Damage} + \text{Tax Revenue}. \tag{12}$$

Based on the welfare function, emission tax is a transfer between firms and the government. That is why in the welfare function, firms' tax expenditures and government's tax revenue cancel out. The tax doesn't have a direct effect on welfare, but it still plays a role because in equilibrium, R&D depends on the emission tax, so changing the emission tax will change R&D, output and pollution.

Under competition, if a regulator is able to commit to an emission tax, in the first stage, the regulator determines the emission tax t to maximize welfare. In the second stage, each firm i determines z_i to maximize its profit π_i. In the third stage, firm i determines output q_i noncooperatively to maximize its profit. Under merger, in the first stage, the regulator determines the emission tax t to maximize welfare. In the second stage, the merged entity determines z_i and z_j to maximize profit π_M. In the third stage, the merged entity determines outputs q_i and q_j to maximize profit π_M. As a result, competition is about competition at all stages, and the merger represents all decisions made to maximize the merged firm's profit.

We also have three scenarios: (1) the regulator is able to commit to an emission tax; (2) the regulator is unable to commit to an emission tax; and (3) the emission tax is exogenous. We are going to make comparisons of equilibrium variables between merger and competition to show the effect of the merger; and comparisons of equilibrium variables between commitment and non-commitment to show the effect of commitment.

4. Commitment

4.1 Competition

We start our analysis from the competition regime and under commitment. In the first stage, the regulator chooses the emission tax t to maximize welfare. In the second stage, each firm i determines z_i to maximize its profit π_i. In the third stage, firm i determines output q_i noncooperatively to maximize its profit.

In Stage 3, each firm chooses its output to maximize its profit.

$$\pi_1 = (a - (q_1 + \theta q_2))q_1 - cq_1 - (q1 - z_1 - \beta z_2)t - \left(\frac{\gamma}{2}\right)z_1^2 \tag{13}$$

$$\pi_2 = (a - (q_2 + \theta q_1))q_2 - cq_2 - (q2 - z_2 - \beta z_1)t - \left(\frac{\gamma}{2}\right)z_2^2 \tag{14}$$

From the first-order conditions, we get equilibrium outputs:[3]

$$q_1 = q_2 = \frac{a - c - t}{2 + \theta} \tag{15}$$

The equilibrium outputs decrease with emission tax, marginal cost, and product differentiation parameter but increase with market size.

In Stage 2, each firm chooses its R&D to maximize its profit. By substituting (15) into (13) and (14) we obtain profit functions for firms.

$$\pi_1 = -\frac{z_1^2\gamma}{2} - \frac{c(a - c - t)}{2 + \theta} - t\left(- z_1 - z_2\beta + \frac{a - c - t}{2 + \theta}\right) + \frac{(a - c - t)\left(a - \frac{(a-c-t)(1+\theta)}{2+\theta}\right)}{2 + \theta} \tag{16}$$

[3]Second-order conditions for maximization are satisfied and proved in Appendix.

$$\pi_2 = -\frac{z_2^2\gamma}{2} - \frac{c(a-c-t)}{2+\theta} - t\left(-z_2 - z_1\beta + \frac{a-c-t}{2+\theta}\right) + \frac{(a-c-t)\left(a-\frac{(a-c-t)(1+\theta)}{2+\theta}\right)}{2+\theta} \tag{17}$$

The first-order conditions engender equilibrium R&D.

$$z_1 = z_2 = \frac{t}{\gamma} \tag{18}$$

R&D increases with the tax (the tax increases the private gain from R&D) and decreases with research costs.

In Stage 1, the regulator determines the emission tax t to maximize welfare. The welfare function has four components: consumer surplus (CS), producer surplus (Profit), environmental damage (D), and tax revenue.

$$C = \left(\frac{1}{2}\right)(q_1^2 + 2\theta q_1 q_2 + q_2^2) \tag{19}$$

$$\text{Profit} = \pi_1 + \pi_2 \tag{20}$$

$$D = \left(\frac{d}{2}\right)((q1 - z_1 - \beta z_2) + q_2 - z_2 - \beta z_1))^2 \tag{21}$$

$$\text{Tax Revenue} = t((q1 - z_1 - \beta z_2) + (q_2 - z_2 - \beta z_1)) \tag{22}$$

$$\begin{aligned} W =& \left(\frac{1}{2}\right)(q_1^2 + 2\theta q_1 q_2 + q_2^2) + \pi_1 + \pi_2 - \left(\left(\frac{d}{2}\right)(q1 - z1 - \beta z_2) + (q_2 - z_2 - \beta z_1))^2\right) \\ &+ t((q1 - z_1 - \beta z_2) + (q_2 - z_2 - \beta z_1)) \end{aligned} \tag{23}$$

By substituting (15) and (18) into (23) we can get welfare in terms of t and other parameters:

$$\begin{aligned} W =& -\frac{t^2}{\gamma} - \frac{2c(a-c-t)}{2+\theta} - \frac{1}{2}\left(\frac{2(a-c-t)^2(1+\theta)}{(2+\theta)^2}\right) + t\left(-\frac{2t(1+\beta)}{\gamma} + \frac{2(a-c-t)}{2+\theta}\right) \\ &- \frac{1}{2}d\left(-\frac{2t(1+\beta)}{\gamma} + \frac{2(a-c-t)}{2+\theta}\right)^2 - 2t\left(-\frac{t(1+\beta)}{\gamma} + \frac{a-c-t}{2+\theta}\right) + \frac{2(a-c-t)\left(a-\frac{(a-c-t)(1+\theta)}{2+\theta}\right)}{2+\theta} \end{aligned} \tag{24}$$

The regulator maximizes W through t by setting $\frac{\partial W}{\partial t} = 0$. Then, the first-order condition yields the equilibrium tax t.

$$t = \frac{(a-c)\gamma(-\gamma + 2d(2+\gamma+\theta+\beta(2+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2 + \gamma(\gamma(1+\theta) + (2+\theta)^2)} \tag{25}$$

As a result, equilibrium values z_1, z_2, q_1, q_2 are:[4]

$$z_1 = z_2 = \frac{(a-c)(-\gamma + 2d(2+\gamma+\theta+\beta(2+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2 + \gamma(\gamma(1+\theta) + (2+\theta)^2)} \tag{26}$$

$$q_1 = q_2 = -\frac{-a + c + \frac{(a-c)\gamma(-\gamma+2d(2+\gamma+\theta+\beta(2+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2+\gamma(\gamma(1+\theta)+(2+\theta)^2)}}{2+\theta} \tag{27}$$

[4]The Appendix provides more details of the results of sections 4.1 and 4.2.

4.2 Merger

If the regulator is able to commit to an emission tax, under merger, in the first stage, the regulator determines the emission tax t to maximize welfare. In the second stage, the merged entity determines z_i and z_j to maximize profit π_M. In the third stage, the merged entity determines outputs q_i and q_j to maximize profit π_M.

Profit is given by:

$$\pi_M = (a-(q_1+\theta q_2))q_1 - cq_1 - (q1 - z1 - \beta z_2)t - \left(\frac{\gamma}{2}\right)z_1^2 + (a-(q_2+\theta q_1))q_2 \\ - cq_2 - (q_2 - z_2 - \beta z_1)t - \left(\frac{\gamma}{2}\right)z_2^2 \tag{28}$$

From the first-order conditions, we get equilibrium output.

$$q_1 = q_2 = \frac{a-c-t}{2(1+\theta)} \tag{29}$$

The equilibrium outputs decrease with emission tax, marginal cost, and product differentiation parameter but increase with market size.

In Stage 2, the merged firm chooses environmental R&D z_1 and z_2 to maximize profit π_M. By substituting (29) into (28) we get profit as a function of R&D:

$$\pi_M = -\frac{\gamma(z_1^2+z_2^2)}{2} - \frac{c(a-c-t)}{1+\theta} - t\left(-z_2 - z_1\beta + \frac{a-c-t}{2(1+\theta)}\right) - t\left(-z_1 - z_2\beta + \frac{a-c-t}{2(1+\theta)}\right) \\ + \frac{(a-c-t)\left(a - \frac{(a-c-t)(1+\theta)}{2(1+\theta)}\right)}{1+\theta} \tag{30}$$

Solving the two first-order conditions $\frac{\partial \pi_M}{\partial z_1} = 0$ and $\frac{\partial \pi_M}{\partial z_2} = 0$ yields equilibrium R&D:

$$z_1 = z_2 = \frac{t+t\beta}{\gamma} \tag{31}$$

R&D increases with the tax and spillovers and decreases with research costs. As spillovers increase, firms internalize R&D, leading to more R&D.

In Stage 1, the regulator determines the emission tax t to maximize welfare. We have four components in the welfare function: consumer surplus (CS), producer surplus (Profit), environmental damage (D), and tax revenue. Functions are identical to (19)-(23).

By substituting (29) and (31) into (23), we can get the welfare function in terms of t and other parameters:

$$W = -\frac{(t+t\beta)^2}{\gamma} - \frac{c(a-c-t)}{1+\theta} + \frac{1}{2}\left(\frac{(a-c-t)^2(1+\theta)}{2(1+\theta)^2}\right) + t\left(\frac{(-2-2\beta)(t+t\beta)}{\gamma} + \frac{a-c-t}{1+\theta}\right) \\ - \frac{1}{2}d\left(\frac{(-2-2\beta)(t+t\beta)}{\gamma} + \frac{a-c-t}{1+\theta}\right)^2 - 2t\left(\frac{(-1-\beta)(t+t\beta)}{\gamma} + \frac{a-c-t}{2(1+\theta)}\right) \\ + \frac{(a-c-t)\left(a - \frac{(a-c-t)(1+\theta)}{2(1+\theta)}\right)}{1+\theta} \tag{32}$$

The regulator maximizes W through t by setting $\frac{\partial W}{\partial t} = 0$. Then, the first-order condition yields the equilibrium tax t:

$$t = \frac{(a-c)\gamma(-\gamma(1+\theta) + 2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2 + \gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))} \tag{33}$$

Therefore, we can solve for equilibrium values of R&D and output:

$$z_1 = z_2 = \frac{(a-c)(1+\beta)(-\gamma(1+\theta) + 2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2 + \gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))} \tag{34}$$

$$q_1 = q_2 = -\frac{-a + c + \frac{(a-c)\gamma(-\gamma(1+\theta)+2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2+\gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))}}{2(1+\theta)} \tag{35}$$

4.3 Comparison

In this section, we analyze the effect of the merger on innovation; therefore, we put numerical values for parameters to compare each variable between merger and competition under commitment. By setting $A = a - c = 1000, \gamma = 100, d = 1$, and $\beta = 0.2$ we consider different levels of product differentiation, θ. It is a common practice for theoretical papers to use numerical simulations with parameter values. The parameter values can ensure positive values for relevant model variables.

Proposition 1:

(1) *When goods are complements ($\theta = -0.5$), the merger increases emission tax, R&D, output, pollution reduction, pollution, and welfare. However, the merger decreases profit.*

(2) *When goods are independent ($\theta = 0$), the merger increases R&D, output, profit, pollution reduction, and welfare. However, the merger decreases emission tax and pollution.*

(3) *When goods are imperfect substitutes ($\theta = 0.5$) or homogenous ($\theta = 1$), the merger increases profit but reduces emission tax, R&D, output, pollution reduction, pollution, and welfare.*

As illustrated by Table 1, when goods are complements, the merger increases output, which leads to higher pollution and higher marginal social damage. Higher marginal social damage leads to a higher emission tax. Since the emission tax is higher under merger, equation (31) implies that $\frac{\partial z_i}{\partial t} = \frac{1+\beta}{\gamma} > 0$, meaning that firms will invest more in R&D. The merged entity internalizes the R&D spillovers, which leads to more R&D. Higher R&D implies a larger pollution reduction under merger. Profit is much lower under merger due to the higher emission tax, higher pollution and higher R&D expenditure.[5] A higher value of welfare is generated under merger due to a higher consumer surplus (due to higher output). If goods are complements, then merger is not profitable because it increases output and emission tax.

As illustrated by Table 2, when goods are independent, the merger increases output and R&D. The merged entity internalizes the R&D spillovers, which leads to more R&D. More R&D means that firms want to reduce their emission tax expenditure, which yields more pollution reduction

[5] Since profit goes down with merger, in this case firms have no incentive to merge.

Table 1

Goods are complements ($\theta = -0.5$) under commitment.

	Merger	Competition
t	**594.305**	399.207
$z_1 = z_2$	**7.13166**	3.99207
$q_1 = q_2$	**405.695**	400.529
Industry Profit	169674.2	323078
Pollution Reduction	**17.116**	9.58096
R&D Spending	**5086.06**	1593.66
Pollution Level, MSD[a]	**794.273**	791.477
Welfare	**408574**	406035

a. MSD represents the marginal social damage.

Table 2

Goods are independent ($\theta = 0$) under commitment.

	Merger	Competition
t	333.159	334.061
$z_1 = z_2$	**3.99791**	3.34061
$q_1 = q_2$	**333.42**	332.97
Industry Profit	**223936**	223300
Pollution Reduction	**9.59498**	8.01746
R&D Spending	**1598.33**	1115.97
Pollution Level, MSD	657.246	657.922
Welfare	**338087**	337524

Table 3

Goods are imperfect substitutes ($\theta = 0.5$) under commitment.

	Merger	Competition
t	154.067	287.707
$z_1 = z_2$	1.84881	2.87707
$q_1 = q_2$	281.978	284.917
Industry Profit	**238876**	163514.4
Pollution Reduction	4.43713	6.90498
R&D Spending	341.808	827.755
Pollution Level, MSD	559.518	562.929
Welfare	287816	288795

Table 4

Goods are homogenous ($\theta = 1$) under commitment.

	Merger	Competition
t	25.7866	253.034
$z_1 = z_2$	0.309439	2.53034
$q_1 = q_2$	243.553	248.989
Industry Profit	**237282**	124887.2
Pollution Reduction	0.742654	6.07281
R&D Spending	9.57527	640.26
Pollution Level, MSD	486.364	491.905
Welfare	250186	252361

and less pollution. Lower pollution leads to lower marginal social damage and a lower tax. Profit is higher under merger due to larger operational profit, lower emission tax and pollution. A higher value of welfare is generated under merger because of higher consumer surplus, higher profit, and lower damage.

As illustrated by tables 3 and 4, when goods are imperfect substitutes or homogenous, the merger increases profit but decreases output. Lower output implies lower pollution. Lower pollution implies a lower value of marginal social damage, which further leads to reductions in the emission tax and R&D. Under merger, lower R&D implies lower pollution reduction. Profit is much higher under merger due to lower emission tax, lower pollution and lower R&D expenditure. As indicated in Figure 1, the merger generates lower welfare, because the merger generates lower consumer surplus, when $\beta = 0.2$, with $\theta = 0.5$ or $\theta = 1$.

We use a set of graphs to illustrate the effect of the merger on the economy under commitment.

There are five parameters in our model: (1) $\beta \in (0,1)$. (2) $\gamma > 0$, which is fixed at 100. (3) $\theta \in (-1,1]$. (4) We fix the damage parameter $d = 1$. (5) Let $A \equiv a - c > 0$, and fix it as $A = 1000$.

We focus on pollution reduction, R&D spending, pollution, and welfare. Under different levels of $\beta \in (0,1), \theta \in (-1,1]$, there is one frontier and two regions in each graph: the white region indicates that the merger increases the value of that variable, and the blue region indicates that the merger decreases the value of that variable, compared with competition. The figures also summarize the effect of $\beta \in (0,1)$ and $\theta \in (-1,1]$, where β is represented on the horizontal axis, and θ is represented on the vertical axis.

If there is no merger, the spillovers only affect pollution, but they do not affect R&D costs or competition between firms. Merger internalizes the spillovers and the merged entity wants to invest more in R&D. As β increases the benefits from the merger become higher compared with competition. For given θ, as β increases, merger increases pollution reduction, R&D spending, and welfare, while decreasing pollution.

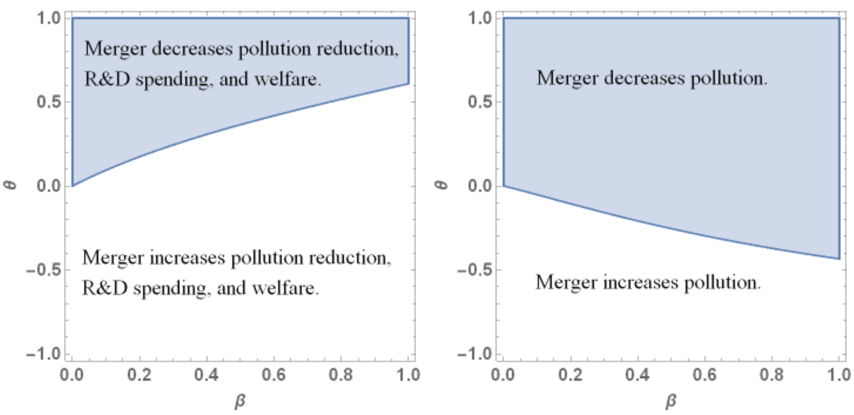

Figure 1. Pollution reduction, R&D spending, pollution, and welfare under commitment.

5. Non-Commitment

5.1 Competition

In this section, we focus on competition under non-commitment. In the first stage, each firm i chooses z_i to maximize its profit π_i. In the second stage, the regulator determines the emission tax t to maximize welfare. In the third stage, firm i chooses output q_i noncooperatively to maximize its profit.

In Stage 3, given the values of R&D and tax in previous stages, firm i determines output q_i noncooperatively to maximize its profit, and we get identical profit equations as (13) and (14), and equilibrium output results as (15).

$$q_1 = q_2 = \frac{a-c-t}{2+\theta} \tag{15}$$

The equilibrium outputs decrease with emission tax, marginal cost, and product differentiation parameter but increase with market size.

In Stage 2, the regulator determines the optimal tax t to maximize welfare (23). By substituting (15) into (23), we get the welfare function in terms of z_1, z_2 and t:

$$\begin{aligned} W =& -\frac{\gamma(z_1^2+z_2^2)}{2} - \frac{2c(a-c-t)}{2+\theta} - \frac{1}{2}\left(\frac{2(a-c-t)^2(1+\theta)}{(2+\theta)^2}\right) + t\left(-(z_1+z_2)(1+\beta) + \frac{2(a-c-t)}{2+\theta}\right) \\ & - \frac{1}{2}d\left(-(z_1+z_2)(1+\beta) + \frac{2(a-c-t)}{2+\theta}\right)^2 - t\left(-z_2 - z_1\beta + \frac{a-c-t}{2+\theta}\right) \\ & - t\left(-z_1 - z_2\beta + \frac{a-c-t}{2+\theta}\right) + \frac{2(a-c-t)\left(a - \frac{(a-c-t)(1+\theta)}{2+\theta}\right)}{2+\theta} \end{aligned} \tag{36}$$

The regulator maximizes W through t by setting $\frac{\partial W}{\partial t} = 0$. Then, the first-order condition yields the equilibrium tax t:

$$t = \frac{c - 2cd + a(-1+2d) - d(z_1+z_2)(1+\beta)(2+\theta)}{1+2d+\theta} \tag{37}$$

The equilibrium tax decreases with spillovers and R&D efforts. Increase in spillovers leads to more R&D, that implies lower marginal social damage, as a result, the equilibrium tax decreases.

In Stage 1, firm i chooses R&D z_i noncooperatively to maximize its profit. By substituting (15) and (37) into (13) and (14), we obtain profit functions in terms of z_1, z_2:

$$\begin{aligned} \pi_1 =& -\frac{z_1^2\gamma}{2} + \frac{c(-a+c-d(z_1+z_2)(1+\beta))}{1+2d+\theta} + \frac{(a-c+d(z_1+z_2)(1+\beta))(2ad+c(1+\theta)-d(z_1+z_2)(1+\beta)(1+\theta))}{(1+2d+\theta)^2} \\ & + \frac{(a-2ad+c(-1+2d)+d(z_1+z_2)(1+\beta)(2+\theta))(a-c+z_1(-1+d(-1+\beta)-\theta)-z_2(d(-1+\beta)+\beta+\beta\theta))}{(1+2d+\theta)^2} \end{aligned} \tag{38}$$

$$\begin{aligned} \pi_2 =& -\frac{z_2^2\gamma}{2} + \frac{c(-a+c-d(z_1+z_2)(1+\beta))}{1+2d+\theta} + \frac{(a-c+d(z_1+z_2)(1+\beta))(2ad+c(1+\theta)-d(z_1+z_2)(1+\beta)(1+\theta))}{(1+2d+\theta)^2} \\ & - \frac{(-a+c+d(z_1-z_2)(-1+\beta)+(z_2+z_1\beta)(1+\theta))(a-2ad+c(-1+2d)+d(z_1+z_2)(1+\beta)(2+\theta))}{(1+2d+\theta)^2} \end{aligned} \tag{39}$$

Solving $\frac{\partial \pi_1}{\partial z_1} = 0$ and $\frac{\partial \pi_2}{\partial z_2} = 0$ yields equilibrium R&D:

$$z_1 = z_2 = \frac{(a-c)(-1-\theta+2d(1+2d+\beta+\theta))}{\gamma(1+\theta)^2 + 2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta) + d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta))} \tag{40}$$

Therefore, we can obtain equilibrium values of emission tax and output:[6]

$$t = \frac{c - 2cd + a(-1+2d) - \frac{2(a-c)d(1+\beta)(2+\theta)(-1-\theta+2d(1+2d+\beta+\theta))}{\gamma(1+\theta)^2+2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta)+d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta))}}{1+2d+\theta} \tag{41}$$

$$q_1 = q_2 = \frac{(a-c)(\gamma+2d((1+\beta)(2+2d+\beta)+\gamma)+(d(1+\beta)(3+\beta)+\gamma)\theta)}{\gamma(1+\theta)^2 + 2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta) + d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta))} \tag{42}$$

5.2 Merger

In the first stage, the merged firm determines z_1 and z_2 to maximize profit π_M. In the second stage, the regulator determines the emission tax t to maximize welfare. In the third stage, the merged firm determines q_1 and q_2 to maximize profit π_M.

In Stage 3, the merged firm chooses outputs to maximize profit π_M. Maximizing (28) over outputs yields:

$$q_1 = q_2 = \frac{a-c-t}{2(1+\theta)} \tag{29}$$

The equilibrium outputs decrease with emission tax, marginal cost, and product differentiation parameter but increase with market size.

In Stage 2, the regulator determines the optimal tax t to maximize the welfare function (23). By substituting (29) into (23), we get welfare in terms of z_1, z_2 and t:

$$\begin{aligned} W = &-\frac{\gamma(z_1^2+z_2^2)}{2} - \frac{c(a-c-t)}{1+\theta} + \frac{1}{2}\left(\frac{(a-c-t)^2(1+\theta)}{2(1+\theta)^2}\right) + t\left(-(z_1+z_2)(1+\beta) + \frac{a-c-t}{1+\theta}\right) \\ &-\frac{1}{2}d\left(-(z_1+z_2)(1+\beta) + \frac{a-c-t}{1+\theta}\right)^2 - t\left(-z_2 - z_1\beta + \frac{a-c-t}{2(1+\theta)}\right) \\ &-t\left(-z_1 - z_2\beta + \frac{a-c-t}{2(1+\theta)}\right) + \frac{(a-c-t)\left(a - \frac{(a-c-t)(1+\theta)}{2(1+\theta)}\right)}{1+\theta} \end{aligned} \tag{43}$$

The regulator maximizes W by setting $\frac{\partial W}{\partial t} = 0$. Solving the first-order condition yields the equilibrium tax t:

$$t = \frac{a(-1+2d-\theta) - 2d(z_1+z_2)(1+\beta)(1+\theta) + c(1-2d+\theta)}{1+2d+\theta} \tag{44}$$

The equilibrium tax decreases with spillovers and R&D efforts.

[6] The Appendix provides more details of the results of sections 5.1 and 5.2.

In Stage 1, the merged firm chooses R&D z_1 and z_2 to maximize profit π_M. By substituting (29) and (44) into (28) we get the profit function in terms of z_1 and z_2:

$$\begin{aligned}\pi_M =& -\frac{z_2^2\gamma}{2} + \frac{c(-a+c-d(z_1+z_2)(1+\beta))}{1+2d+\theta} + \frac{(a-c+d(z_1+z_2)(1+\beta))(2ad+c(1+\theta)-d(z_1+z_2)(1+\beta)(1+\theta))}{(1+2d+\theta)^2}\\ &- \frac{(-a+c+d(z_1-z_2)(-1+\beta)+(z_2+z_1\beta)(1+\theta))(c(-1+2d-\theta)+2d(z_1+z_2)(1+\beta)(1+\theta)+a(1-2d+\theta))}{(1+2d+\theta)^2}\\ &+ \frac{1}{2(1+2d+\theta)^2}(-2c(a-c+d(z_1+z_2)(1+\beta))(1+2d+\theta) - z_1^2\gamma(1+2d+\theta)^2\\ &+2(c(-1+2d-\theta)+2(a-c+d(z_1+z_2)(1+\beta))(2ad+c(1+\theta)-d(z_1+z_2)(1+\beta)(1+\theta))\\ &+2d(z_1+z_2)(1+\beta)(1+\theta)+a(1-2d+\theta))(a-c+z_1(-1+d(-1+\beta)-\theta)\\ &-z_2(d(-1+\beta)+\beta+\beta\theta)))\end{aligned} \tag{45}$$

Solving $\frac{\partial\pi_M}{\partial z_1} = 0$ and $\frac{\partial\pi_M}{\partial z_2} = 0$ yields equilibrium R&D:

$$z_1 = z_2 = \frac{(a-c)(1+\beta)(4d^2+4d(1+\theta)-(1+\theta)^2)}{\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))} \tag{46}$$

As a result, equilibrium values of the emission tax and output are:

$$t = \frac{a(-1+2d-\theta)+c(1-2d+\theta) - \frac{4(a-c)d(1+\beta)^2(1+\theta)(4d^2+4d(1+\theta)-(1+\theta)^2)}{\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))}}{1+2d+\theta} \tag{47}$$

$$q_1 = q_2 = -\frac{-a+c+\frac{a(-1+2d-\theta)+c(1-2d+\theta) - \frac{4(a-c)d(1+\beta)^2(1+\theta)(4d^2+4d(1+\theta)-(1+\theta)^2)}{\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))}}{1+2d+\theta}}{2(1+\theta)} \tag{48}$$

5.3 Comparison

We use a set of tables to illustrate the effect of the merger on innovation and other variables under non-commitment. We fix $A = 1000$, $\gamma = 100$, $d = 1$ and $\beta = 0.2$ and allow θ to change.

Proposition 2: *Under non-commitment, the merger increases R&D, output, pollution reduction, and welfare. However, the merger decreases pollution. The merger increases profit and decreases the emission tax except if goods are complements.*

As illustrated by Table 5, when goods are complements, if there is a merger, R&D, emission tax, output, and welfare increase. The merged entity internalizes the R&D spillovers, which leads to more R&D. Higher R&D leads to more pollution reduction. The merger reduces pollution, marginal social damage, and profit. A higher value of pollution reduction leads to lower pollution, followed by lower marginal social damage. Profit is much lower under merger due to the higher emission tax and higher R&D expenditure. The merger increases welfare due to larger consumer surplus and lower damage.[7] If goods are complements, then merger is not profitable because it increases output and emission tax.

[7] Since profit decreases with the merger, in this case firms have no incentive to merge.

Table 5

Goods are complements ($\theta = -0.5$) under non-commitment.

	Merger	Competition
$z_1 = z_2$	**10.8895**	7.7331
t	**589.546**	388.864
$q_1 = q2$	**410.454**	407.424
Industry Profit	172022	333226
Pollution Reduction	**26.1347**	18.5594
R&D Spending	**11858**	5980.08
Pollution Level, MSD	794.773	796.288
Welfare	**408981**	408833

Table 6

Goods are independent ($\theta = 0$) under non-commitment.

	Merger	Competition
$z_1 = z_2$	**9.10036**	5.88697
t	318.773	323.914
$q_1 = q_2$	**340.614**	338.043
Industry Profit	**230716**	229656
Pollution Reduction	**21.8409**	14.1287
R&D Spending	**8281.66**	3465.64
Pollution Level, MSD	659.386	661.957
Welfare	**339533**	339254

Table 7

Goods are imperfect substitutes ($\theta = 0.5$) under non-commitment.

	Merger	Competition
$z_1 = z_2$	**7.33323**	4.7093
t	127.772	277.641
$q_1 = q_2$	**290.743**	288.944
Industry Profit	**250466**	167897
Pollution Reduction	**17.5997**	11.3023
R&D Spending	**5377.62**	2217.75
Pollution Level, MSD	563.886	566.585
Welfare	**290327**	289928

As illustrated by tables 6 and 7, when goods are independent or imperfect substitutes, since the government chooses the emission tax after R&D decisions made by firms, equation (44) implies that $\frac{\partial t}{\partial z_i} = \frac{-2d(1+\beta)(1+\theta)}{1+2d+\theta} < 0$, therefore a merged firm has incentives to increase R&D in the first stage. The merger increases R&D, output, and welfare. The merged entity internalizes the R&D spillovers, which leads to more R&D. Moreover, higher R&D implies higher pollution reduction. If higher pollution reduction decreases pollution, then marginal social damage decreases as well. Profit is higher under merger due to larger operational profit, lower pollution and lower emission tax. The merger has a lower tax compared with competition for several reasons. There are two opposite effects on the emission tax. The first effect is the total pollution-reducing effect which leads to emission tax increases; the government increases the emission tax to control pollution, and as a result, total pollution goes down. The second effect is the production-increasing effect, which leads to emission tax decreases. By using an emission tax, the firm's emission ex-

Table 8
Goods are homogenous ($\theta = 1$) under non-commitment.

	Merger	Competition
$z_1 = z_2$	**5.75153**	3.90168
t	-13.8037	242.977
$q_1 = q_2$	**253.451**	252.341
Industry Profit	**253450**	128105
Pollution Reduction	**13.8037**	9.36403
R&D Spending	**3308.01**	1522.31
Pollution Level, MSD	493.098	495.318
Welfare	**253546**	253138

penditure would increase, and, as a result, firms would lower their outputs to minimize emission expenditures. However, the government wants more outputs, therefore by reducing the emission tax, the government could encourage more production by firms; the pollution-reducing effect is dominated by the production-increasing effect in this case. The merger enhances welfare because of higher values of consumer surplus and profit, and a lower value of damage.

As illustrated by Table 8, when goods are homogenous, normally the merger will reduce output, but the government wants more production, so the government indirectly subsidizes production by using a negative emission tax (i.e., emission subsidy) to encourage production and make production even more profitable (i.e., the pollution-reducing effect is dominated by the production-increasing effect). As in Ouchida and Goto (2014), the economic intuition behind an emission subsidy is as follows: an emission subsidy is equivalent to the policy combination of a production subsidy and an abatement tax since a firm's emission function is defined as pollution generated from production minus abatements generated from R&D. A production subsidy has two effects: a damage-increasing effect and decreasing effect of market inefficiency. When the damage parameter is sufficiently small, the damage-increasing effect is dominated by the improvement effect in market inefficiency. In our model, we have a small damage parameter and in the scenario of a non-commitment emission tax policy towards Cournot duopolists, by using part (ii) of Proposition 1 from Ouchida and Goto (2014), we illustrate the result of a negative emission tax in equilibrium. Therefore, output goes up after the merger. The merger increases R&D, profit, and welfare. The merged entity internalizes R&D spillovers, which leads to more R&D. Higher R&D implies a larger pollution reduction, which leads to lower pollution. Lower pollution implies lower marginal social damage. Profit is much higher under merger due to a negative emission tax and lower pollution. The merger always enhances welfare because of higher consumer surplus and profit, and lower damage.

Figure 2 shows that, for any value of spillover and product differentiation parameters under non-commitment, the merger always increases pollution reduction, R&D spending and decreases pollution. The merger always increases welfare except in the small area shown above, in which the merger decreases welfare. In this small area, due to lower profit, the merger generates lower welfare compared with competition.

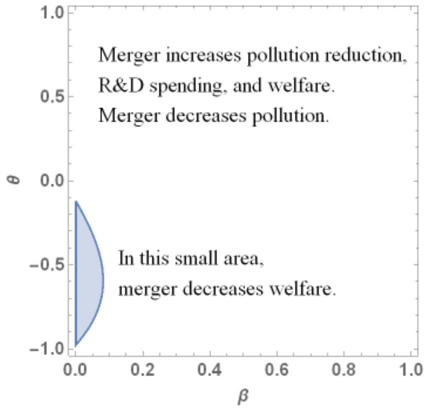

Figure 2. Pollution reduction, R&D spending, pollution, and welfare under non-commitment.

6. Exogenous Tax Scenario

This case is relevant when the tax is set based on country-wide considerations or on an international agreement, and hence is not responsive to the specific conditions of the industry under study. In this section we focus mostly on a positive emission tax, since in the real world the problem seems more often to be how to cut emissions, rather than how to encourage them. Thus, assume $t > 0$.

6.1 Competition

If the emission tax is exogenous, in competition, we have equilibrium values of R&D and output:[8]

$$z_1 = z_2 = \frac{t}{\gamma} \tag{49}$$

R&D increases with the tax and decreases with research costs.

$$q_1 = q_2 = \frac{a - c - t}{2 + \theta} \tag{50}$$

The equilibrium outputs decrease with emission tax, marginal cost, and product differentiation parameter but increase with market size.

6.2 Merger

If the emission tax is exogenous, the merger generates the following equilibrium values:

$$z_1 = z_2 = \frac{t(1 + \beta)}{\gamma} \tag{51}$$

R&D increases with the tax and spillovers and decreases with research costs.

$$q_1 = q_2 = \frac{a - c - t}{2(1 + \theta)} \tag{52}$$

[8] The Appendix provides more details on the derivation of the results of sections 6.1 and 6.2.

The equilibrium outputs decrease with emission tax, marginal cost, and product differentiation parameter but increase with market size.

The following proposition compares R&D, output, R&D spending and pollution reduction between competition and merger.

Proposition 3: *With an exogenous $t > 0$:*

(1) *The merger increases R&D and pollution reduction iff $\beta > 0$, and has no effect on them iff $\beta = 0$.*

(2) *The merger increases output iff $\theta < 0$;*

(3) *The merger decreases output iff $\theta > 0$;*

(4) *The merger leaves output unchanged iff $\theta = 0$.*

Proof. See the Appendix.

The intuition for these results will be provided in the following subsection.

6.3 Comparison

To understand these results, we use numerical values for parameters to compare each variable between merger and competition. We set the emission tax at $t = 20$ along with previous assumptions that $A = 1000$, $\gamma = 100$, $d = 1$, $\beta = 0.2$, and analyze different values of product differentiation parameter, θ.

Proposition 4: *Under a positive exogenous emission tax:*

(1) *When goods are complements, the merger increases R&D, output, profit, pollution reduction, pollution and marginal social damage, but reduces welfare.*

(2) *When goods are independent, the merger increases R&D, pollution reduction, and welfare. Output and profit stay the same between the two regimes. The merger reduces pollution and marginal social damage compared with competition.*

(3) *When goods are imperfect substitutes or homogenous, the merger increases R&D, profit, pollution reduction, and welfare. The merger reduces output, pollution and marginal social damage.*

As illustrated by Table 9, when goods are complements and emission tax is exogenous, the merger increases output. A firm has incentives to reduce pollution tax expenditure, therefore more R&D is generated. The merged entity internalizes R&D spillovers, which leads to more R&D. More R&D implies more pollution reduction. Under merger, more output leads to more pollution, and higher pollution implies higher marginal social damage. The merger increases profit due to higher operational profit. As in Figure 3, if goods are complements, the merger greatly decreases welfare due to larger environmental damage.

As illustrated by Table 10, when goods are independent and the emission tax is exogenous, the merged firm has incentives to internalize R&D spillovers and invest more in R&D in the first stage

Table 9
Goods are complements ($\theta = -0.5$) under exogenous tax scenario.

	Merger	Competition
$z_1 = z_2$	**0.24**	0.2
$q_1 = q_2$	**980**	653.333
Industry Profit	**960406**	853694
Pollution Reduction	**0.576**	0.48
R&D Spending	**5.76**	4
Pollution Level, MSD	**1959.42**	1306.19
Welfare	-439877	240179

Table 10
Goods are independent ($\theta = 0$) under exogenous tax scenario.

	Merger	Competition
$z_1 = z_2$	**0.24**	0.2
$q_1 = q_2$	490	490
Industry Profit	480206	480206
Pollution Reduction	**0.576**	0.48
R&D Spending	**5.76**	4
Pollution Level, MSD	979.424	979.52
Welfare	**260259**	260166

Table 11
Goods are imperfect substitutes ($\theta = 0.5$) under exogenous tax scenario.

	Merger	Competition
$z_1 = z_2$	**0.24**	0.2
$q_1 = q_2$	326.667	392
Industry Profit	**320140**	307334
Pollution Reduction	**0.576**	0.48
R&D Spending	**5.76**	4
Pollution Level, MSD	652.757	783.52
Welfare	**280215**	246548

Table 12
Goods are homogenous ($\theta = 1$) under exogenous tax scenario.

	Merger	Competition
$z_1 = z_2$	**0.24**	0.2
$q_1 = q_2$	245	326.67
Industry Profit	**240106**	213428
Pollution Reduction	**0.576**	0.48
R&D Spending	**5.76**	4
Pollution Level, MSD	489.424	652.853
Welfare	**250176**	226798

to reduce emission tax expenditure. The merger increases R&D, which implies more pollution reduction. Equilibrium output stays the same between merger and competition. Pollution is lower under merger because of a higher value of pollution reduction, and lower pollution implies lower marginal social damage. Interestingly, the merger generates similar profit to competition, due to larger R&D under merger: on the one hand, a larger R&D would reduce the emission tax expenditure, but on the other hand, it involves more R&D expenditure. As a result, the two effects offset each other. As in Figure 3, if goods are independent, the merger increases welfare due to less environmental damage compared with competition.[9]

As illustrated by tables 11 and 12, when goods are imperfect substitutes or homogenous, the merged firm wants to reduce pollution expenditure; as a result, the merger internalizes R&D spillovers and generates more R&D in the first stage. More R&D implies more pollution reduction. The equilibrium quantity is lower under merger compared with competition. The merger reduces pollution, due to higher R&D and lower ouput; as a result, the merger generates less marginal social damage. Profit is much higher under merger due to higher operational profit and lower pollution compared with competition. As in Figure 3, the merger increases welfare because of higher profits and lower environmental damage.

Figure 3 shows that if the exogenous emission tax is positive (negative), for any value of spillovers, the merger always increases (decreases) pollution reduction. Moreover, the merger always increases R&D spending. For any value of spillovers, as products become more similar, the merger decreases pollution and increases welfare.

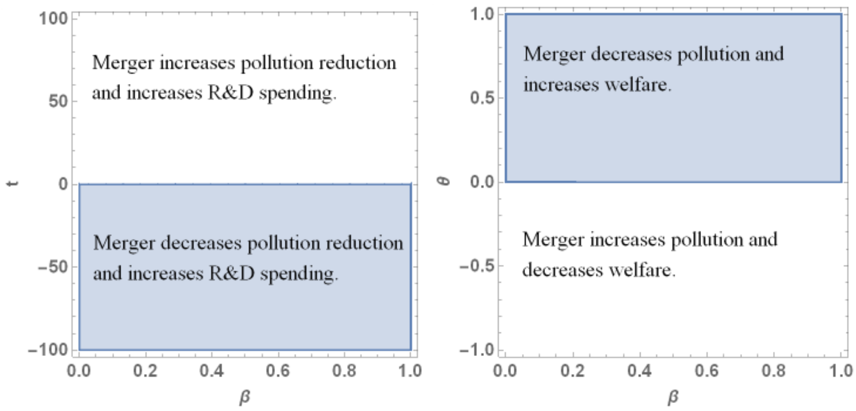

Figure 3. Pollution reduction, R&D spending, pollution, and welfare under exogenous tax scenario.

7. Effect of Commitment

Within each regime of merger and competition, we compare variables between the scenarios of commitment and non-commitment. We hold the original assumptions of $A = 1000$, $\gamma = 100$, $d = 1$, $\beta = 0.2$. Since tables share the same pattern that commitment only increases the emission tax and decreases other variables compared with non-commitment, tables are suppressed in this section.

Proposition 5: *For any type of goods, under any regime, commitment generates a higher emission tax, and lower R&D, output, profit, pollution reduction, pollution, and welfare compared with non-commitment.*

[9]Since profit decreases with merger, in this case firms have no incentive to merge.

For any type of goods, under any regime under commitment, since the emission tax is generated in the first stage, the model structure leads to a higher value of the emission tax. In the second stage, R&D depends on the emission tax only. Equilibrium values of R&D in (18) and (31) show that $\frac{\partial z_i}{\partial t} > 0$, which implies that a higher tax yields higher R&D in the second stage. However, it is still less than the non-commitment value of R&D. In the third stage, optimal output depends on the emission tax only. Equilibrium values of output in (15) and (29) show that $\frac{\partial q_i}{\partial t} < 0$, which implies that a higher tax yields a lower output in the third stage. Lower R&D under commitment leads to less pollution reduction. Under commitment, a small value of output leads to lower pollution. Profit is lower under commitment due to lower operational profit and higher emission tax. Commitment generates lower consumer surplus and lower profit. These components contribute to lower welfare compared with non-commitment.

8. The Merger-R&D and Commitment-R&D Relationship

In this section, we emphasize two important relationships. The first one is the effect of the merger on R&D. The second relationship is the effect of commitment on R&D.

Proposition 6:

(1) *Merger has a positive effect on R&D under non-commitment and the exogenous tax scenario.*

(2) *Under commitment, if goods are imperfect substitutes or homogenous, merger has a negative effect on R&D; if goods are complements or independent, merger has a positive effect on R&D.*

(3) *For any types of goods under any regime, commitment has a negative effect on R&D.*

Table 13 summarizes the relationship between merger and R&D as predicted by the model. The rows represent different levels of product differentiation, while the columns represent different scenarios. We compare merger's R&D and competition's R&D under different circumstances. As the table suggests, in the model studied here, the sign of the relationship is more often positive than negative. However, the results do suggest there exist two cases where the relationship is negative: if goods are imperfect substitutes or homogenous under commitment.

Table 14 summarizes the relationship between commitment and R&D as predicted by the model. The rows represent different levels of product differentiation, while the columns represent different regimes. We compare commitment's and non-commitment's R&D under different circumstances. As the table suggests, in the model studied here, the sign of the relationship is always negative. For any type of goods under any regime, the results suggest a negative relationship between commitment and R&D.

In our setting, non-commitment yields superior innovation performance to commitment, irrespective of product differentiation, market structure (merger versus competition), and R&D spillovers. The intuition for this result is the following. Under non-commitment, firms anticipate that high R&D spending will lead to lower marginal social damage, and hence a lower emission

Table 13
The effect of merger on R&D.

	Commitment	Non-commitment	Exogenous emission tax
θ = -0.5	+	+	+
$\theta = 0$	+	+	+
$\theta = 0.5$	-	+	+
$\theta = 1$	-	+	+

Table 14
The effect of commitment on R&D.

	Merger	Competition
θ = -0.5	-	-
$\theta = 0$	-	-
$\theta = 0.5$	-	-
$\theta = 1$	-	-

tax. This provides an added incentive for firms to invest more in R&D, to reduce the emission tax.

Under commitment, the government chooses the tax before firms choose R&D. Thus, at the time of investing in R&D, firms take the tax as given, and do not invest more strategically to induce a lower marginal social damage and thus a lower emission tax. It is true that the government, as a leader under commitment, has the incentive to increase the tax in the first stage to increase R&D by firms in the second stage. However, this effect is weaker than the additional incentive of firms under non-commitment, whence the superiority of the latter over commitment. Moreover, for the government, tax revenues ultimately don't matter, since they increase government revenue but reduce profits.

9. Conclusion

This paper explores the effect of merger on environmental R&D. In our model, under the policy regimes of merger and competition, firms play a three-stage game by investing in R&D to reduce the carbon emission expenditure. We consider the fact that product differentiation can play a role into the environmental policy design and analyze different policy implementations under the regulator's full commitment, no commitment and exogenous emission tax. We study the relationship between merger and R&D and the relationship between commitment and R&D. We also evaluate the overall effect of merger and the overall effect of commitment.

As the main results, proposition 6 suggests that: (1) Merger has a positive effect on R&D under non-commitment and the exogenous tax scenarios. (2) Under commitment, if goods are imperfect substitutes or homogenous, merger has a negative effect on R&D; if goods are complements or independent, merger has a positive effect on R&D. (3) For any types of goods under any regime, commitment has a negative effect on R&D. Propositions 1-5 compare equilibrium variables between merger and competition. (4) Under commitment, if products are highly differentiated, the tax from the merger is higher. As products become more and more similar, the tax becomes lower under the merger. R&D, output, pollution reduction, pollution, marginal social damage, and welfare follow a similar trend. However, the profit from the merger would increase as products become more and more similar. (5) Under non-commitment, as products become less differentiated, the tax becomes lower under merger. However, profit becomes higher under merger. R&D, output, pollution reduction, and welfare are always higher under merger. Pollution and marginal social damage are always lower under merger. (6) Under the exogenous tax scenario, as products become less differentiated, output, pollution, and marginal social damage become less under merger, and welfare increases with the merger. R&D and pollution reduction are always higher under merger. (7) If we compare commitment to non-commitment: for any type of goods, under any regime, commitment generates a higher emission tax, and lower R&D, output, profit, pollution reduction, pollution, marginal social damage, and welfare.

In our paper, there are three aspects which are different from previous studies. First, our research focuses on total merger, which is cooperation at all stages, especially in the output stage. Previously, studies only focused on R&D cooperation. Another difference is that our study fully considers product differentiation. This is different from other studies which only focus on homogenous products. Specifically, we consider complements, independent goods, imperfect substitutes, and homogenous goods. Third, our research includes three different scenarios: commitment, non-commitment, and exogenous tax. Other major studies only focus on one or two of them (e.g. Poyago-Theotoky, 1999, 2007, 2010; Petrakis and Poyago-Theotoky, 2002; Ouchida and Goto, 2011, 2014, 2016a,b). In short, we study full mergers with product differentiation, and study combinations of different policy regimes and scenarios simultaneously.

The results indicate that merger will be profitable for firms, increase R&D investment and welfare (1) if goods are independent under commitment; (2) if goods are independent, imperfect substitutes, or homogeneous under non-commitment; and (3) if goods are imperfect substitutes or homogenous under the exogenous tax scenario. Also, non-commitment will encourage R&D investment, increase profit, and enhance welfare, compared with commitment for any type of goods and under any regime. As a result, public policies should focus on the encouragement of mergers and promote the regulator's non-commitment ability if the market structure satisfies the above conditions, since doing so would not only lead firms to engage in more R&D investments in terms of increasing innovation outputs as pollution reductions, but it also would have the beneficial effect of increasing the firms' profit and welfare.

There are many dimensions in which this research line can be extended. First, our results have been obtained in the context of a duopolistic market. For future research, it seems very promising to extend the analysis to an n-firm oligopoly. Second, under the setting with more than two firms, we would have to consider merger encompassing less than the total number of firms in the market and examine the role of insider and outsider firms and how the interplay of these types of firms affects equilibrium variables.

Appendix

Commitment Scenario - Competition Regime

In Subgame Perfect Nash Equilibrium, the equilibrium values of R&D spending, pollution reduction, pollution, and welfare are:

$$\text{R\&D spending} = \frac{(a-c)^2\gamma(\gamma-2d(2+\gamma+\theta+\beta(2+\theta)))^2}{(2d(2+\gamma+\theta+\beta(2+\theta))^2+\gamma(\gamma(1+\theta)+(2+\theta)^2))^2}$$

$$\text{Pollution Reduction} = \frac{2(a-c)(1+\beta)(-\gamma+2d(2+\gamma+\theta+\beta(2+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2+\gamma(\gamma(1+\theta)+(2+\theta)^2)}$$

$$\text{Pollution} = \frac{2(a-c)\gamma(3+\beta+\gamma+\theta)}{2d(2+\gamma+\theta+\beta(2+\theta))^2+\gamma(\gamma(1+\theta)+(2+\theta)^2)}$$

$$\text{Welfare} = \frac{(a-c)^2(\gamma(3+\gamma+\theta)+2d(3+\gamma+\theta+\beta^2(3+\theta)+2\beta(3+\gamma+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2+\gamma(\gamma(1+\theta)+(2+\theta)^2)}$$

Commitment Scenario - Merger Regime

In Subgame Perfect Nash Equilibrium, equilibrium values of R&D spending, pollution reduction, pollution and welfare are:

$$\text{R\&D spending} = \frac{(a-c)^2(1+\beta)^2\gamma(\gamma(1+\theta)-2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))^2}{(2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2+\gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta)))^2}$$

$$\text{Pollution Reduction} = \frac{2(a-c)(1+\beta)^2(-\gamma(1+\theta)+2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2+\gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))}$$

$$\text{Pollution} = \frac{2(a-c)\gamma(3+\gamma+3\theta+3\beta(2+\beta)(1+\theta))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2+\gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))}$$

$$\text{Welfare} = \frac{(a-c)^2(2d(1+\beta)^2+\gamma)(3+\gamma+3\theta+3\beta(2+\beta)(1+\theta))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2+\gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))}$$

Consider the level of product differentiation $\theta = 0$, we generate some analytical results from these special cases. Let the superscript C denote variables under competition, and M denote variables under merger.

(1) R&D spending^M > R&D spending^C iff

$$-\frac{A^2\gamma(\gamma-2d(2+2\beta+\gamma))^2}{(\gamma(4+\gamma)+2d(2+2\beta+\gamma)^2)^2}+\frac{A^2(1+\beta)^2\gamma(\gamma-2d(2+2\beta(2+\beta)+\gamma))^2}{(2d(2+2\beta(2+\beta)+\gamma)^2+\gamma(4+4\beta(2+\beta)+\gamma))^2}>0$$

(2) Pollution Reduction^M > Pollution Reduction^C iff

$$-\frac{2A(1+\beta)(-\gamma+2d(2+2\beta+\gamma))}{\gamma(4+\gamma)+2d(2+2\beta+\gamma)^2}+\frac{2A(1+\beta)^2(-\gamma+2d(2+2\beta(2+\beta)+\gamma))}{2d(2+2\beta(2+\beta)+\gamma)^2+\gamma(4+4\beta(2+\beta)+\gamma)}>0$$

(3) Pollution^M > Pollution^C iff

$$-\frac{2A\gamma(3+\beta+\gamma)}{\gamma(4+\gamma)+2d(2+2\beta+\gamma)^2}+\frac{2A\gamma(3+3\beta(2+\beta)+\gamma)}{2d(2+2\beta(2+\beta)+\gamma)^2+\gamma(4+4\beta(2+\beta)+\gamma)}>0$$

Non-Commitment Scenario - Competition Regime

In the Subgame Perfect Nash Equilibrium, equilibrium values of R&D spending, pollution reduction, and pollution are:[10]

$$\text{R\&D spending} = \frac{(a-c)^2\gamma(-1-\theta+2d(1+2d+\beta+\theta))^2}{(\gamma(1+\theta)^2+2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta)+d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta)))^2}$$

$$\text{Pollution Reduction} = \frac{2(a-c)(1+\beta)(-1-\theta+2d(1+2d+\beta+\theta))}{\gamma(1+\theta)^2+2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta)+d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta))}$$

$$\text{Pollution} = \frac{2(a-c)(2d(1+\beta+\gamma)+d(1+\beta)^2\theta+(1+\beta+\gamma)(1+\theta))}{\gamma(1+\theta)^2+2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta)+d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta))}$$

Non-Commitment Scenario - Merger Regime

In the Subgame Perfect Nash Equilibrium, the equilibrium values of R&D spending, pollution reduction, and pollution are:

$$\text{R\&D spending} = \frac{(a-c)^2(1+\beta)^2\gamma(-4d^2-4d(1+\theta)+(1+\theta)^2)^2}{(\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))^2}$$

$$\text{Pollution Reduction} = \frac{2(a-c)(1+\beta)^2(4d^2+4d(1+\theta)-(1+\theta)^2)}{\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))}$$

$$\text{Pollution} = \frac{2(a-c)(1+2d+\theta)(1+\gamma+\theta+\beta(2+\beta)(1+\theta))}{\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))}$$

Consider the level of product differentiation $\theta = 0$, we generate some analytical results from these special cases.[11]

(1) R&D spending^M > R&D spending^C iff

$$\frac{A^2(1-4d-4d^2)^2(1+\beta)^2\gamma}{(\gamma+4d(2+2\beta(2+\beta)+\gamma)+4d^2(2+2\beta(2+\beta)+\gamma))^2}-\frac{A^2(-1+2d(1+2d+\beta))^2\gamma}{(\gamma+4d^2(2+2\beta+\gamma)+d(6+2\beta(4+\beta)+4\gamma))^2}>0$$

(2) Pollution Reduction^M > Pollution Reduction^C iff

$$\frac{2A(-1+4d+4d^2)(1+\beta)^2}{\gamma+4d(2+2\beta(2+\beta)+\gamma)+4d^2(2+2\beta(2+\beta)+\gamma)}-\frac{2A(1+\beta)(-1+2d(1+2d+\beta))}{\gamma+4d^2(2+2\beta+\gamma)+d(6+2\beta(4+\beta)+4\gamma)}>0$$

(3) Pollution^M > Pollution^C iff

$$\frac{2A(1+2d)(1+\beta(2+\beta)+\gamma)}{\gamma+4d(2+2\beta(2+\beta)+\gamma)+4d^2(2+2\beta(2+\beta)+\gamma)}-\frac{2A(1+\beta+\gamma+2d(1+\beta+\gamma))}{\gamma+4d^2(2+2\beta+\gamma)+d(6+2\beta(4+\beta)+4\gamma)}>0$$

[10] The expression for welfare is not shown, due to its length.

[11] Let the superscript C denote variables under competition, and M denote variables under merger.

Exogenous Tax Scenario - Competition Regime

If the emission tax is exogenous, there are two stages under competition: in stage 1, firm i chooses R&D non-cooperatively and maximizes its profit. In stage 2, firm i determines output non-cooperatively to maximize its profit. We have the same profit functions as (13) and (14), solving the first-order conditions $\frac{\partial \pi_1}{\partial q_1} = 0$ and $\frac{\partial \pi_2}{\partial q_2} = 0$ yields equilibrium values of output in stage 2:

$$q_1 = q_2 = \frac{a-c-t}{2+\theta} \tag{15}$$

In stage 1, by substituting (15) into profit functions (13) and (14), we generate profit functions (16) and (17) in terms of z_1 and z_2:

$$\pi_1 = -\frac{z_1^2\gamma}{2} - \frac{c(a-c-t)}{2+\theta} - t\left(-z_1 - z_2\beta + \frac{a-c-t}{2+\theta}\right) + \frac{(a-c-t)\left(a - \frac{(a-c-t)(1+\theta)}{2+\theta}\right)}{2+\theta}$$

$$\pi_2 = -\frac{z_2^2\gamma}{2} - \frac{c(a-c-t)}{2+\theta} - t\left(-z_2 - z_1\beta + \frac{a-c-t}{2+\theta}\right) + \frac{(a-c-t)\left(a - \frac{(a-c-t)(1+\theta)}{2+\theta}\right)}{2+\theta}$$

Solving the first-order conditions $\frac{\partial \pi_1}{\partial z_1} = 0$ and $\frac{\partial \pi_2}{\partial z_2} = 0$ yields equilibrium R&D, which is the same as equation (18).

As a conclusion, if the emission tax is exogenous, in competition, we have equilibrium values of R&D, output, R&D spending, pollution reduction, pollution and welfare:

$z_1 = z_2 = \frac{t}{\gamma}$

$q_1 = q_2 = \frac{a-c-t}{2+\theta}$

R&D spending $= \frac{t^2}{\gamma}$

Pollution reduction $= \frac{2t(1+\beta)}{\gamma}$

Pollution $= \frac{-2t(1+\beta)}{\gamma} + \frac{2(a-c-t)}{2+\theta}$

Welfare $= -\frac{\gamma(2(a-c)t\gamma - (a-c)^2\gamma(3+\theta) + t^2(4+\gamma+(4+\gamma)\theta+\theta^2)) + 2d((-a+c)\gamma + t(2+\gamma+\theta+\beta(2+\theta)))^2}{\gamma^2(2+\theta)^2}$

Exogenous Tax Scenario - Merger Regime

If the emission tax is exogenous, there are two stages under merger: in stage 1, the merged firm chooses R&D z_1 and z_2 to maximize profit π_M. In stage 2, the merged firm determines outputs q_1 and q_2 to maximize profit π_M. We have the same profit function as (28). Solving the first-order conditions $\frac{\partial(\pi_M)}{\partial q_1} = 0$ and $\frac{\partial(\pi_M)}{\partial q_2} = 0$ yields equilibrium values of output in stage 2:

$$q_1 = q_2 = \frac{a-c-t}{2(1+\theta)} \tag{29}$$

In stage 1, by substituting (29) into profit π_M, we get the profit function (30) in terms of z_1 and z_2:

$$\pi_M = -\frac{\gamma(z_1^2+z_2^2)}{2} - \frac{c(a-c-t)}{1+\theta} - t\left(-z_2 - z_1\beta + \frac{a-c-t}{2(1+\theta)}\right) - t\left(-z_1 - z_2\beta + \frac{a-c-t}{2(1+\theta)}\right) + \frac{(a-c-t)\left(a - \frac{(a-c-t)(1+\theta)}{2(1+\theta)}\right)}{1+\theta}$$

Solving the first-order conditions $\frac{\partial(\pi_M)}{\partial z_1} = 0$ and $\frac{\partial(\pi_M)}{\partial z_2} = 0$ yields equilibrium R&D.

As a conclusion, if the emission tax is exogenous, the merger generates the following equilibrium values of R&D, output, pollution reduction, pollution and welfare:

$z_1 = z_2 = \frac{t(1+\beta)}{\gamma}$

$q_1 = q_2 = \frac{a-c-t}{2(1+\theta)}$

R&D spending $= \frac{(t+t\beta)^2}{\gamma}$

Pollution reduction $= \frac{(2+2\beta)(t+t\beta)}{\gamma}$

Pollution $= \frac{(-2-2\beta)(t+t\beta)}{\gamma} + \frac{a-c-t}{1+\theta}$

Welfare $= -\frac{2d((-a+c)\gamma+t(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))^2+\gamma(1+\theta)(-3(a-c)^2\gamma+2(a-c)t\gamma+t^2(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta)))}{4\gamma^2(1+\theta)^2}$

Proof of Proposition 3

i) $z^C = \frac{t}{\gamma}; z^M = \frac{t(\beta+1)}{\gamma}$

$z^M - z^C = \frac{t\beta}{\gamma}$

$z^M > z^C$ iff $\beta > 0$

$z^M = z^C$ iff $\beta = 0$

Let S denote R&D spending.

$S^C = \frac{t^2}{\gamma}; S^M = \frac{(t+t\beta)^2}{\gamma}$

$S^M - S^C = \frac{t^2\beta(\beta+2)}{\gamma}$

$S^M > S^C$ iff $\beta(\beta+2) > 0$

$S^M = S^C$ iff $\beta = 0$

ii) $q^C = \frac{a-c-t}{2+\theta}; q^M = \frac{a-c-t}{2(1+\theta)}$

$q^M > q^C$ iff $2(1+\theta) < 2+\theta$

$\theta < 0$

$q^M < q^C$ iff $\theta > 0$

$q^M = q^C$ iff $\theta = 0$

Derivation - Consumer Surplus

The representative consumer's utility function is

$$U(q_1, q_2) = a(q_1 + q_2) - \left(\frac{1}{2}\right)(q_1^2 + 2\theta q_1 q_2 + q_2^2)$$

The consumer's utility function does not include pollution effects.
The representative consumer chooses outputs q_1 and q_2 to maximize

$$U(q_1, q_2) - p_1 q_1 - p_2 q_2$$

The first-order conditions are

$$a - q_1 - \theta q_2 - p_1 = 0$$

$$a - q_2 - \theta q_1 - p_2 = 0$$

The inverse demand functions are

$$p_1(q_1, q_2) = a - (q_1 + \theta q_2)$$

$$p_2(q_1, q_2) = a - (q_2 + \theta q_1)$$

Substituting the inverse demand functions for p_1, p_2, we can obtain consumer surplus:

$$\begin{aligned}
CS(q_1, q_2) &= U(q_1, q_2) - p_1(q_1, q_2)q_1 - p_2(q_1, q_2)q_2 \\
&= U(q_1, q_2) - aq_1 + (q_1^2 + \theta q_2 q_1) - aq_2 + (q_2^2 + \theta q_1 q_2) \\
&= -\left(\frac{1}{2}\right)(q_1^2 + 2\theta q_1 q_2 + q_2^2) + (q_1^2 + \theta q_2 q_1) + (q_2^2 + \theta q_1 q_2) \\
&= -\left(\frac{1}{2}\right)(q_1^2 + q_2^2) - \theta q_1 q_2 + (q_1^2 + q_2^2) + (2\theta q_1 q_2) \\
&= \left(\frac{1}{2}\right)(q_1^2 + q_2^2) + \theta q_1 q_2 \\
&= \left(\frac{1}{2}\right)(q_1^2 + 2\theta q_1 q_2 + q_2^2)
\end{aligned}$$

Second-Order Conditions - Commitment - Competition

By setting $A = a - c = 1000$, $\gamma = 100$, $d = 1$, and $\beta = 0.2$ we consider different levels of product differentiation, θ.

The second-order conditions of output for maximization are

$$\frac{\partial^2 \pi_1}{\partial q_1^2} = -2$$

$$\frac{\partial^2 \pi_2}{\partial q_2^2} = -2$$

The second-order conditions of R&D for maximization are

$$\frac{\partial^2 \pi_1}{\partial z_1^2} = -100$$

$$\frac{\partial^2 \pi_2}{\partial z_2^2} = -100$$

The second-order conditions of tax for maximization is

$$\frac{\partial^2 W}{\partial t^2} = -2.3068 \qquad (\theta = -0.5)$$

$$\frac{\partial^2 W}{\partial t^2} = -1.5686 \qquad (\theta = 0)$$

$$\frac{\partial^2 W}{\partial t^2} = -1.17898 \qquad (\theta = 0.5)$$

$$\frac{\partial^2 W}{\partial t^2} = -0.941465 \qquad (\theta = 1)$$

Second-Order Conditions - Commitment - Merger

By setting $A = a - c = 1000$, $\gamma = 100$, $d = 1$, and $\beta = 0.2$ we consider different levels of product differentiation, θ.

The second-order conditions of output for maximization are

$$\frac{\partial^2 \pi_M}{\partial q_1^2} = -2$$

$$\frac{\partial^2 \pi_M}{\partial q_2^2} = -2$$

The second-order conditions of R&D for maximization are

$$\frac{\partial^2 \pi_M}{\partial z_1^2} = -100$$

$$\frac{\partial^2 \pi_M}{\partial z_2^2} = -100$$

The second-order conditions of tax for maximization is

$$\frac{\partial^2 W}{\partial t^2} = -5.14483 \qquad (\theta = -0.5)$$

$$\frac{\partial^2 W}{\partial t^2} = -1.58723 \qquad (\theta = 0)$$

$$\frac{\partial^2 W}{\partial t^2} = -0.845807 \qquad (\theta = 0.5)$$

$$\frac{\partial^2 W}{\partial t^2} = -0.558429 \qquad (\theta = 1)$$

Second-Order Conditions - Non-Commitment - Competition

By setting $A = a - c = 1000$, $\gamma = 100$, $d = 1$, and $\beta = 0.2$ we consider different levels of product differentiation, θ.

The second-order conditions of output for maximization are

$$\frac{\partial^2 \pi_1}{\partial q_1^2} = -2$$

$$\frac{\partial^2 \pi_2}{\partial q_2^2} = -2$$

The second-order conditions of tax for maximization is

$$\frac{\partial^2 W}{\partial t^2} = -2.2222 \quad (\theta = -0.5)$$
$$\frac{\partial^2 W}{\partial t^2} = -1.5 \quad (\theta = 0)$$
$$\frac{\partial^2 W}{\partial t^2} = -1.12 \quad (\theta = 0.5)$$
$$\frac{\partial^2 W}{\partial t^2} = -0.8889 \quad (\theta = 1)$$

The second-order conditions of R&D for maximization are

$$\frac{\partial^2 \pi_1}{\partial z_1^2} = -100.979 \quad (\theta = -0.5)$$
$$\frac{\partial^2 \pi_2}{\partial z_2^2} = -100.979 \quad (\theta = -0.5)$$
$$\frac{\partial^2 \pi_1}{\partial z_1^2} = -101.28 \quad (\theta = 0)$$
$$\frac{\partial^2 \pi_2}{\partial z_2^2} = -101.28 \quad (\theta = 0)$$
$$\frac{\partial^2 \pi_1}{\partial z_1^2} = -101.479 \quad (\theta = 0.5)$$
$$\frac{\partial^2 \pi_2}{\partial z_2^2} = -101.479 \quad (\theta = 0.5)$$
$$\frac{\partial^2 \pi_1}{\partial z_1^2} = -101.62 \quad (\theta = 1)$$
$$\frac{\partial^2 \pi_2}{\partial z_2^2} = -101.62 \quad (\theta = 1)$$

Second-Order Conditions - Non-Commitment - Merger

By setting $A = a - c = 1000$, $\gamma = 100$, $d = 1$, and $\beta = 0.2$ we consider different levels of product differentiation, θ.

The second-order conditions of output for maximization are

$$\frac{\partial^2 \pi_M}{\partial q_1^2} = -2$$
$$\frac{\partial^2 \pi_M}{\partial q_2^2} = -2$$

The second-order conditions of tax for maximization is

$$\frac{\partial^2 W}{\partial t^2} = -5 \quad (\theta = -0.5)$$
$$\frac{\partial^2 W}{\partial t^2} = -1.5 \quad (\theta = 0)$$
$$\frac{\partial^2 W}{\partial t^2} = -0.7778 \quad (\theta = 0.5)$$
$$\frac{\partial^2 W}{\partial t^2} = -0.5 \quad (\theta = 1)$$

The second-order conditions of R&D for maximization are

$$\frac{\partial^2 \pi_M}{\partial z_1^2} = -100.691 \qquad (\theta = -0.5)$$

$$\frac{\partial^2 \pi_M}{\partial z_2^2} = -100.691 \qquad (\theta = -0.5)$$

$$\frac{\partial^2 \pi_M}{\partial z_1^2} = -101.28 \qquad (\theta = 0)$$

$$\frac{\partial^2 \pi_M}{\partial z_2^2} = -101.28 \qquad (\theta = 0)$$

$$\frac{\partial^2 \pi_M}{\partial z_1^2} = -101.763 \qquad (\theta = 0.5)$$

$$\frac{\partial^2 \pi_M}{\partial z_2^2} = -101.763 \qquad (\theta = 0.5)$$

$$\frac{\partial^2 \pi_M}{\partial z_1^2} = -102.16 \qquad (\theta = 1)$$

$$\frac{\partial^2 \pi_M}{\partial z_2^2} = -102.16 \qquad (\theta = 1)$$

Second-Order Conditions - Exogenous Tax - Competition

By setting $A = a - c = 1000$, $\gamma = 100$, $d = 1$, $\beta = 0.2$, and $t = 20$ we consider different levels of product differentiation, θ.

The second-order conditions of output for maximization are

$$\frac{\partial^2 \pi_1}{\partial q_1^2} = -2$$

$$\frac{\partial^2 \pi_2}{\partial q_2^2} = -2$$

The second-order conditions of R&D for maximization are

$$\frac{\partial^2 \pi_1}{\partial z_1^2} = -100$$

$$\frac{\partial^2 \pi_2}{\partial z_2^2} = -100$$

Second-Order Conditions - Exogenous Tax - Merger

By setting $A = a - c = 1000$, $\gamma = 100$, $d = 1$, $\beta = 0.2$, and $t = 20$ we consider different levels of product differentiation, θ.

The second-order conditions of output for maximization are

$$\frac{\partial^2 \pi_M}{\partial q_1^2} = -2$$

$$\frac{\partial^2 \pi_M}{\partial q_2^2} = -2$$

The second-order conditions of R&D for maximization are

$$\frac{\partial^2 \pi_M}{\partial z_1^2} = -100$$

$$\frac{\partial^2 \pi_M}{\partial z_2^2} = -100$$

Commitment Scenario - Competition Regime

$$z_1 = z_2 = \frac{(a-c)(-\gamma + 2d(2+\gamma+\theta+\beta(2+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2 + \gamma(\gamma(1+\theta)+(2+\theta)^2)} > 0 \quad \text{iff}$$

$$(a-c)(-\gamma + 2d(2+\gamma+\theta+\beta(2+\theta))) > 0$$

$$2d(2+\gamma+\theta+\beta(2+\theta))^2 + \gamma(\gamma(1+\theta)+(2+\theta)^2) > 0;$$

$$q_1 = q_2 = -\frac{-a + c + \frac{(a-c)\gamma(-\gamma+2d(2+\gamma+\theta+\beta(2+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2+\gamma(\gamma(1+\theta)+(2+\theta)^2)}}{2+\theta} > 0 \quad \text{iff}$$

$$c + \frac{(a-c)\gamma(-\gamma + 2d(2+\gamma+\theta+\beta(2+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2 + \gamma(\gamma(1+\theta)+(2+\theta)^2)} < a$$

$$2+\theta > 0;$$

$$\text{Pollution Reduction} = \frac{2(a-c)(1+\beta)(-\gamma + 2d(2+\gamma+\theta+\beta(2+\theta)))}{2d(2+\gamma+\theta+\beta(2+\theta))^2 + \gamma(\gamma(1+\theta)+(2+\theta)^2)} > 0 \quad \text{iff}$$

$$(a-c)(1+\beta)(-\gamma + 2d(2+\gamma+\theta+\beta(2+\theta))) > 0$$

$$2d(2+\gamma+\theta+\beta(2+\theta))^2 + \gamma(\gamma(1+\theta)+(2+\theta)^2) > 0.$$

Commitment Scenario - Merger Regime

$$z_1 = z_2 = \frac{(a-c)(1+\beta)(-\gamma(1+\theta) + 2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2 + \gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))} > 0 \quad \text{iff}$$

$$(a-c)(1+\beta)(-\gamma(1+\theta) + 2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))) > 0$$

$$2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2 + \gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta)) > 0;$$

$$q_1 = q_2 = -\frac{-a + c + \frac{(a-c)\gamma(-\gamma(1+\theta)+2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2+\gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))}}{2(1+\theta)} > 0 \quad \text{iff}$$

$$(a-c)(2d(1+\beta)^2+\gamma)(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))(2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2 + \gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))) > 0$$

$$1+\theta > 0;$$

$$\text{Pollution Reduction} = \frac{2(a-c)(1+\beta)^2(-\gamma(1+\theta) + 2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)))}{2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2 + \gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta))} > 0 \quad \text{iff}$$

$$(a-c)(1+\beta)^2(-\gamma(1+\theta) + 2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))) > 0$$

$$2d(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))^2 + \gamma(1+\theta)(4+\gamma+4\theta+4\beta(2+\beta)(1+\theta)) > 0.$$

Non-Commitment Scenario - Competition Regime

$$z_1 = z_2 = \frac{(a-c)(-1-\theta+2d(1+2d+\beta+\theta))}{\gamma(1+\theta)^2 + 2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta) + d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta))} > 0 \quad \text{iff}$$

$$(a-c)(-1-\theta+2d(1+2d+\beta+\theta)) > 0$$

$$\gamma(1+\theta)^2 + 2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta) + d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta)) > 0;$$

$$q_1 = q_2 = \frac{(a-c)(\gamma + 2d((1+\beta)(2+2d+\beta)+\gamma) + (d(1+\beta)(3+\beta)+\gamma)\theta)}{\gamma(1+\theta)^2 + 2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta) + d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta))} > 0 \quad \text{iff}$$

$$(a-c)(\gamma + 2d((1+\beta)(2+2d+\beta)+\gamma) + (d(1+\beta)(3+\beta)+\gamma)\theta) > 0$$

$$\gamma(1+\theta)^2 + 2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta) + d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta)) > 0;$$

$$\text{Pollution Reduction} = \frac{2(a-c)(1+\beta)(-1-\theta+2d(1+2d+\beta+\theta))}{\gamma(1+\theta)^2+2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta)+d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta))} > 0 \quad \text{iff}$$

$$(a-c)(1+\beta)(-1-\theta+2d(1+2d+\beta+\theta)) > 0$$

$$\gamma(1+\theta)^2+2d^2(2(2+2\beta+\gamma)+(1+\beta)(3+\beta)\theta)+d(1+\theta)(6+4\gamma+3\theta+\beta(4+\beta)(2+\theta)) > 0.$$

Non-Commitment Scenario - Merger Regime

$$z_1 = z_2 = \frac{(a-c)(1+\beta)(4d^2+4d(1+\theta)-(1+\theta)^2)}{\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))} > 0 \quad \text{iff}$$

$$(a-c)(1+\beta)(4d^2+4d(1+\theta)-(1+\theta)^2) > 0$$

$$\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)) > 0;$$

$$q_1 = q_2 = -\frac{-a+c+\dfrac{a(-1+2d-\theta)+c(1-2d+\theta)-\frac{4(a-c)d(1+\beta)^2(1+\theta)(4d^2+4d(1+\theta)-(1+\theta)^2)}{\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))}}{1+2d+\theta}}{2(1+\theta)} > 0 \quad \text{iff}$$

$$(a-c)(1+\theta)(\gamma+2d((3+2d)(1+\beta)^2+\gamma)+(6d(1+\beta)^2+\gamma)\theta)(\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)) \\ +4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))) > 0$$

$$1+\theta > 0;$$

$$\text{Pollution Reduction} = \frac{2(a-c)(1+\beta)^2(4d^2+4d(1+\theta)-(1+\theta)^2)}{\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))} > 0 \quad \text{iff}$$

$$(a-c)(1+\beta)^2(4d^2+4d(1+\theta)-(1+\theta)^2) > 0$$

$$\gamma(1+\theta)^2+4d^2(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta))+4d(1+\theta)(2+\gamma+2\theta+2\beta(2+\beta)(1+\theta)) > 0.$$

Exogenous Tax Scenario - Competition Regime

$$z_1 = z_2 = \frac{t}{\gamma} > 0 \quad \text{iff}$$

$$t > 0$$

$$\gamma > 0;$$

$$q_1 = q_2 = \frac{a-c-t}{2+\theta} > 0 \quad \text{iff}$$

$$-a+c+t < 0$$

$$-2-\theta < 0;$$

$$\text{Pollution reduction} = \frac{2t(1+\beta)}{\gamma} > 0 \quad \text{iff}$$

$$2t+2t\beta > 0$$

$$\gamma > 0.$$

Exogenous Tax Scenario - Merger Regime

$z_1 = z_2 = \dfrac{t(1+\beta)}{\gamma} > 0 \quad \text{iff}$

$t + t\beta > 0$

$\gamma > 0;$

$q_1 = q_2 = \dfrac{a-c-t}{2(1+\theta)} > 0 \quad \text{iff}$

$-a + c + t < 0$

$1 + \theta > 0;$

Pollution reduction $= \dfrac{(2+2\beta)(t+t\beta)}{\gamma} > 0 \quad \text{iff}$

$t(1+\beta)^2 > 0$

$\gamma > 0.$

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