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# Balancing Infrastructure and Human Capital: Optimal Fiscal Composition for Sustainable Growth

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### Abstract

This paper investigates the relationship between fiscal policy composition and long-run economic growth by extending the classic Alesina–Rodrik framework. We develop a dynamic model that distinguishes between capital-augmenting public investments (such as infrastructure) and labor-enhancing expenditures (including human capital development), both financed through a wealth tax. Our central hypothesis is that an optimal allocation of public spending exists which maximizes the net return on capital and thereby supports sustained growth. However, political pressures—stemming from heterogeneous factor endowments and median voter preferences—can drive fiscal policies away from this efficiency benchmark, leading to suboptimal tax rates and spending compositions that may even trigger growth traps. By employing comparative statics and equilibrium analysis, we demonstrate how redistributive forces influence the choice of fiscal instruments, ultimately affecting aggregate productivity and capital accumulation. The findings offer novel theoretical insights into the trade-offs between redistribution and growth, underscoring the critical importance of aligning fiscal composition with underlying production technologies to achieve both efficient and politically feasible outcomes.

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#### Introduction 1.

We study how the composition of fiscal policy—in particular, the allocation of public spending between capital-augmenting (infrastructure) and labor-enhancing (human capital) investments—affects long-run economic growth. Our central hypothesis is that there exists an optimal balance of public expenditure that maximizes the net return on capital, thereby supporting higher sustained growth. In contrast, when political pressures drive fiscal policy away from this efficiency benchmark, suboptimal tax rates and expenditure allocations may impede growth and even lead to stagnation traps.

Our contribution is twofold. First, we extend the well-known Alesina–Rodrik framework by introducing a dual public goods model. This extension allows us to distinguish explicitly between the growth effects of infrastructure and human capital investments financed by a wealth tax. Second, by incorporating political economy elements—such as heterogeneous factor endowments and median voter preferences—we show how redistributive pressures can force the actual fiscal policy away from the growth-maximizing configuration. This analysis not only provides new theoretical insights into the trade-offs between redistribution and growth but also offers a formal mechanism linking fiscal composition to long-run performance.

The importance of this contribution lies in its ability to bridge gaps between growth theory, public economics, and political economy. Whereas earlier literature has separately documented the adverse effects of excessive redistribution and the benefits of targeted public investments, our model integrates these strands to demonstrate how fiscal policy design must reconcile efficiency with political feasibility.

Methodologically, our approach relies on constructing a dynamic model built upon a Cobb-Douglas production function that features both private inputs and two types of governmentprovided public goods. We analyze the equilibrium conditions under a balanced-growth assumption and use comparative statics to explore how variations in the wealth tax rate and the allocation parameter affect factor returns and growth. Although our analysis is theoretical, it is motivated by empirical regularities observed in income distribution, political preferences, and public expenditure outcomes.

The remainder of the paper is organized as follows. Section 2 provides a review of the literature on fiscal policy, redistribution, and growth, emphasizing both classical and recent developments. Section 3 introduces our dual public goods model and details its key assumptions and mechanisms. Section 4 presents the equilibrium analysis and comparative statics that yield the optimal fiscal policy configuration, while Section 5 discusses the political economy implications and potential growth traps arising from misaligned fiscal choices. Finally, Section 6 concludes by summarizing our findings, acknowledging the model's limitations, and suggesting avenues for future research.

#### 2. Literature Review

Research in political economy has explored the interrelations among distributive politics, fiscal policy composition, and economic growth. Work by Alesina and Rodrik (1994), for instance,



established that the way governments choose taxation and spending policies is deeply intertwined with the underlying distribution of wealth. Their framework indicated that higher wealth inequality generates political pressures leading to more redistributive fiscal policies, which, in turn, may have adverse effects on long-run economic growth by distorting capital accumulation.

Building on this, subsequent research broadened these perspectives. Persson and Tabellini (2002) extended the analysis by incorporating more refined measurements of political institutions, highlighting the central role of the median voter's preferences in determining the size and composition of government. Their work reinforces the idea that democratization tends to favor redistributive policies, although the growth consequences depend on the trade-offs inherent in taxation.

Empirical evidence has also informed these theoretical predictions. Meltzer and Richard (1981)'s work on the rational theory of government size provided early support for the notion that the median voter's income position is a key determinant of redistributive policy. Later studies, such as Olson (1993), further explored distinctions between democratic and authoritarian regimes, demonstrating that the institutional context shapes both the extent and the nature of redistribution. More recently, Acemoglu et al. (2015) examined how democratic institutions interact with redistributive policies to affect inequality. Their analysis underscores that the impact of political rights on fiscal policy is multifaceted and that the specific design of redistributive measures matters greatly for aggregate economic performance.

Parallel to this, research on financial markets and inequality has offered complementary insights. Essays by Blaum (2012) provided evidence that imperfections in financial intermediation may exacerbate the misallocation of resources when inequality is high. Such studies suggest that inefficient financial markets can magnify the adverse effects of redistributive policies on growth, creating a channel through which inequality not only prompts political redistribution but also undermines efficient capital allocation.

Recent developments have also broadened the analysis of intergenerational persistence and inequality. Several theories examine how family investment decisions, skill formation, and social interactions contribute to the reproduction of inequality. Classic family investment models by Becker and Tomes (1979) and Becker and Tomes (1986) highlight how parental income constrains investments in children's education, while Loury (1981) demonstrated that differences in initial endowments can have enduring effects on income distribution. Building on these, work by Galor and Zeira (1993) and Han and Mulligan (2001) incorporated borrowing constraints and credit market imperfections, generating nonlinearities in the intergenerational transmission process. Social models, such as the work of Durlauf (1996b) and Durlauf (1996a), have focused on neighborhood effects and social capital, showing how segregation can amplify disparities. Political economy contributions also emphasize how redistributive politics are affected not only by median voter preferences (Meltzer and Richard, 1981; Alesina and Rodrik, 1994) but also by power differentials, with studies by Acemoglu and Robinson (2008) and Roemer (2006) demonstrating how elite control might skew policy. Finally, research incorporating psychological dimensions like aspirations (Ray, 2006; Genicot and Ray, 2017) suggests individual expectations also mediate the inequality-mobility relationship.



Further examining how fiscal policy influences long-run growth, Barro (1990), in an early endogenous-growth model, showed that productive spending financed by a flat tax can raise output up to a point, establishing an inverted-U link between growth and "useful" government services. The idea that distributional conflict can push tax rates beyond that optimum, as introduced by Alesina and Rodrik (1994), suggests that greater wealth inequality can raise political demand for capital taxation and depress investment. Subsequent political-economy models, including the common-pool framework of Battaglini and Coate (2008), indicate that legislatures might systematically bias budgets toward targeted transfers when voters are heterogeneous, potentially crowding out growth-enhancing outlays. Meta-regressions, such as the synthesis of 67 studies by Bom and Lightart (2014), confirm that the productive component matters, finding a notable long-run output elasticity of public capital, albeit with high heterogeneity. Efficiency often appears to dominate quantity, a conclusion reinforced by IMF frontier estimates suggesting that closing public-investment efficiency gaps could significantly lift world GDP (Kapsoli et al., 2023). However, a review by Ansar et al. (2016) of Chinese mega-projects reveals that many can destroy value, underscoring the danger of "white-elephant" investment when project selection is politicized.

Human-capital expenditures also display high, distinct returns. Synthesizing international test data, Hanushek and Woessmann (2020) concluded that a one-standard-deviation increase in cognitive skills is associated with a roughly two-percentage-point rise in steady-state GDP per capita. Methodological advances have been important here. The financing side, however, cannot be ignored. Using Danish administrative records, Jakobsen et al. (2020) identified a nearly one-for-one fall in private wealth following increases in recurrent net-wealth taxation. OECD comparative studies (OCDE, 2018) reach similar conclusions, warning that wealth taxes may reduce capital formation unless paired with very high investment efficiency.

The literature thus indicates a robust link between inequality, political institutions, and economic growth. On one hand, models such as those by Alesina and Rodrik (1994) and Persson and Tabellini (2002) suggest that greater inequality tends to increase the demand for redistribution in democracies. On the other hand, studies like Acemoglu et al. (2015) and Blaum (2012) warn that excessive redistribution, especially when combined with inefficient public spending or financial market frictions, can impede growth. The present paper extends this literature by introducing a dual public goods framework. This framework aims to disentangle the effects of wealth taxation on capital versus labor, thereby providing new insights into the political economy trade-offs that underpin fiscal policy choices regarding its composition and their impact on long-run economic performance.

#### Model 3.

We extend the Alesina-Rodrik framework by introducing two distinct categories of productive public goods, enabling an analysis of fiscal policy that incorporates both tax levels and expenditure composition. This extension preserves the AK growth structure while enriching its policy relevance, as governments face inherent tradeoffs when allocating limited resources bet-



ween infrastructure development and labor-enhancing services. The model captures how these compositional choices influence long-term growth trajectories through their effects on capital productivity and workforce efficiency.

## 3.1 Production Structure with Dual Public Inputs

Firms operate under a Cobb-Douglas technology that combines private capital  $K_{j,t}$ , labor  $L_{j,t}$ , and two government-provided inputs:

$$Y_{j,t} = AK_{j,t}^{\alpha}G_{1,t}^{\beta}(G_{2,t}L_{j,t})^{1-\alpha-\beta}, \quad 0 < \alpha, \beta < 1, \quad \alpha + \beta < 1, \quad j = 1, 2, \dots, m,$$

Here,  $G_{1,t}$  represents capital-augmenting infrastructure such as transportation networks or research facilities, while  $G_{2,t}$  denotes labor-enhancing services like education and healthcare. The parameter  $\alpha$  governs private capital's direct contribution to output,  $\beta$  determines infrastructure's effectiveness in boosting capital productivity, and  $1-\alpha-\beta$  captures labor's responsiveness to human capital improvements. We assume that total labor supply L is fixed and exogenously given, with no population growth, so that  $L_t = L^1$  for all t. The restriction  $\alpha + \beta < 1$  ensures diminishing returns to private inputs while maintaining positive labor shares, reflecting empirical evidence on factor income distribution. This structure generalizes the original Alesina-Rodrik specification by allowing simultaneous analysis of infrastructure and human capital investments.

The government levies a wealth tax  $\tau_t$  on aggregate capital stock  $K_t$ , allocating revenues between the two public goods through a composition parameter  $\mu$ :

$$G_{1,t} = \mu \tau_t K_t$$
,  $G_{2,t} = (1 - \mu) \tau_t K_t$ ,  $\mu \in (0,1)$ 

The tax rate  $\tau_t$  endogenously adjusts to satisfy national accounting consistency, creating a feed-back loop between fiscal policy and economic fundamentals. Higher values of  $\mu$  prioritize infrastructure development at the expense of labor-enhancing services, mimicking growth strategies seen in rapidly industrializing economies. Conversely, lower  $\mu$  values emphasize human capital formation, characteristic of social-democratic welfare states.

The government operates under a balanced budget constraint, financing public goods entirely through wealth taxation on aggregate private capital. Let  $V_t \equiv K_t$  represent total financial wealth in this closed economy with no government debt or natural resource endowments. The fiscal rule becomes:

$$G_{1,t} + G_{2,t} = \mu \tau_t K_t + (1 - \mu) \tau_t K_t = \tau_t K_t$$

This specification clarifies that  $\tau_t$  constitutes a wealth tax (levied on capital stock  $K_t$ ), distinct from a capital income tax. A common mislabeling in the literature (including Alesina-Rodrik's original work) erroneously refers to  $\tau_t$  as an income tax. The distinction proves crucial: a wealth tax reduces capital returns through subtraction  $(r_t - \tau_t)K_t$ , whereas an income tax would multiplicatively reduce returns  $(1 - \tau_t)r_tK_t$ . Our formulation adheres strictly to wealth taxation to maintain analytical coherence.

<sup>&</sup>lt;sup>1</sup>Without loss of generality and to eliminate the classic scale-effect (see Barro and Sala-i Martin (2004, pp. 218–220)), we normalize aggregate labor supply to L = 1.



Physical capital evolves according to:

$$\dot{K}_t = Y_t - C_t - (G_{1,t} + G_{2,t}) = Y_t - C_t - \tau_t K_t$$

where we follow Alesina-Rodrik in abstracting from depreciation ( $\delta = 0$ ) to focus on fiscal policy effects. This simplification allows clearer isolation of tax impacts on growth dynamics. The term  $\tau_t K_t$  represents both the government's revenue and the resource cost of public goods provision.

While some models conceptualize  $K_t$  as "broad capital" incorporating human capital, we strictly interpret it as physical capital (machinery, buildings, infrastructure). This concrete specification: Avoids aggregation challenges between physical and human capital; Clarifies the wealth tax base as tangible assets; and Maintains consistency with national accounts practice.

The exclusion of human capital from  $K_t$  implies labor-enhancing public goods  $G_{2,t}$  operate through wage effects rather than capital valuation, aligning with empirical observations that education investments primarily boost labor productivity rather than asset prices.

#### 3.2 Firm Optimization and Factor Demand

Firms maximize profits in competitive markets, treating public goods  $(G_{1,t}, G_{2,t})$  and factor prices  $(r_t, w_t)$  as exogenous. Each firm solves:

$$\max_{K_{j,t},L_{j,t}} \pi_{j,t} = AK_{j,t}^{\alpha} G_{1,t}^{\beta} (G_{2,t}L_{j,t})^{1-\alpha-\beta} - r_t K_{j,t} - w_t L_{j,t}$$

The solution characterizes how firms translate public goods into productive efficiency while responding to market prices. The first-order condition equates capital's marginal product to its rental rate.

$$r_t = \alpha \frac{Y_{j,t}}{K_{j,t}}$$

The rental rate  $r_t$  absorbs  $\alpha$  % of firm j's output per unit capital. Infrastructure  $G_{1,t}$  amplifies this return through  $\beta$ , as better roads/ports reduce effective transport costs, while labor-enhancing  $G_{2,t}$  allows more efficient capital deployment. Meanwhile, labor demand satisfies wage = marginal product:

$$w_t = (1 - \alpha - \beta) \frac{Y_{j,t}}{L_{j,t}}$$

Wages capture  $(1 - \alpha - \beta)$  % of output per worker. Public education  $G_{2,t}$  directly elevates wages by raising labor productivity, a 1% increase in  $G_{2,t}$  boosts  $w_t$  by  $(1 - \alpha - \beta)$ %, ceteris paribus. Infrastructure  $G_{1,t}$  indirectly affects wages through capital deepening, as better infrastructure attracts private investment.

Under perfect competition, all firms adopt identical capital-labor ratios  $K_{j,t}/L_{j,t} = K_t/L$ . Aggregating across firms by substituting  $G_{1,t} = \mu \tau_t K_t$  and  $G_{2,t} = (1 - \mu)\tau_t K_t$ :

$$Y_t = A\mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} \tau_t^{1 - \alpha} L^{1 - \alpha - \beta} K_t$$

This economy-wide production function maintains constant returns to capital  $(Y_t \propto K_t)$ , preserving the AK growth structure. The equilibrium factor prices become:

$$r_t = \alpha \frac{Y_t}{K_t}, \quad w_t = (1 - \alpha - \beta) \frac{Y_t}{L}$$



Factor payments exhaust  $(\alpha + 1 - \alpha - \beta) = (1 - \beta)\%$  of output, with the residual  $\beta\%$  financing public goods through taxation. This residual reflects infrastructure's role as a quasi-fixed factor - while firms don't pay directly for  $G_{1,t}$ , its provision requires diverting resources from private consumption.

### 3.3 Equilibrium Factor Prices and Fiscal Policy Linkages

In symmetric equilibrium, all firms adopt identical capital-labor ratios  $K_{j,t}/L_{j,t} = K_t/L$ . The equilibrium interest rate is (see Appendix A.1)<sup>2</sup>

$$r(\tau,\mu) = \alpha A \left(\mu^{\beta} (1-\mu)^{1-\alpha-\beta}\right) \tau^{1-\alpha} L^{1-\alpha-\beta}, \quad \text{with } \frac{\partial r}{\partial \tau} > 0, \quad \frac{\partial r}{\partial \mu} \ge 0 \text{ if } \mu \le \frac{\beta}{1-\alpha}. \tag{1}$$

The positive response of r to  $\tau$  reflects infrastructure's role as a capital complement—higher taxation expands public capital services, elevating capital's marginal productivity. The non-monotonic effect of  $\mu$  arises from competing margins: reallocating taxes toward infrastructure  $(G_1)$  boosts capital productivity when  $\mu$  is below  $\beta/(1-\alpha)$ , but beyond this threshold, excessive  $G_1$  crowds out labor-enhancing  $G_2$ , dampening aggregate efficiency.

The real wage equals labor's marginal product. Differentiating the production function with respect to  $L_{j,t}$ . Expressing wages in per capita terms  $(w_t = \omega(\tau, \mu)K_t)$ :

$$\omega(\tau,\mu) = (1 - \alpha - \beta)A\left(\mu^{\beta}(1 - \mu)^{1 - \alpha - \beta}\right)\tau^{1 - \alpha}L^{-(\alpha + \beta)}, \quad \frac{\partial\omega}{\partial\tau} > 0, \quad \frac{\partial\omega}{\partial\mu} \ge 0 \text{ if } \mu \le \frac{\beta}{1 - \alpha}.$$
 (2)

While higher  $\tau$  uniformly raises wages by expanding both public goods, the composition  $\mu$  dictates labor's relative gain. When  $\mu < \beta/1 - \alpha$ , prioritizing labor-enhancing  $G_2$  amplifies human capital's productivity, lifting wages. Conversely, when  $\mu > \beta/(1-\alpha)$ , reallocating fiscal spending toward infrastructure reduces overall efficiency, lowering both wages and capital returns. The relative distributional impact depends on the magnitude of these changes and the initial factor shares.

In the extended Alesina–Rodrik framework where firms produce according to

$$Y_{j,t} = A K_{j,t}^{\alpha} (G_{1,t})^{\beta} (G_{2,t} L_{j,t})^{1-\alpha-\beta}, \quad 0 < \alpha, \beta < 1, \ \alpha + \beta < 1,$$

and the government finances the two public goods by a wealth tax  $\tau$  on aggregate capital according to

$$G_{1,t} = \mu \tau K_t$$
 and  $G_{2,t} = (1 - \mu) \tau K_t$ ,

the equilibrium gross rental rate of capital is given by

$$r(\tau,\mu) = \alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{1-\alpha} L^{1-\alpha-\beta}.$$

The unique value of the public spending composition that maximizes the gross rental rate (and hence the net return on capital and overall growth) is

$$\mu^* = \frac{\beta}{1 - \alpha}.$$

<sup>&</sup>lt;sup>2</sup>Since  $\alpha + \beta < 1$ , we have  $0 < \beta < 1 - \alpha$   $\Longrightarrow$   $0 < \frac{\beta}{1-\alpha} < 1$ , and hence  $\mu^* = \frac{\beta}{1-\alpha} \in (0,1)$ .



Figure 1: Relationship Between Interest Rate, Wages and Public Goods Allocation

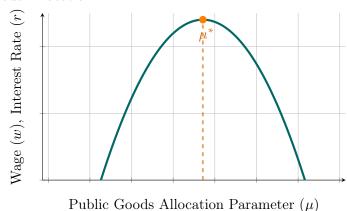


Figure 1. Theoretical non-monotonic relationship between wages and public goods allocation. The curve peaks at  $\mu^* = \beta/(1-\alpha)$ , showing the optimal balance between infrastructure and human capital investments. Parameters:  $\alpha = 0.3$ ,  $\beta = 0.2$ , A = 1,  $\tau = 0.2$ , L = 1.

In production technology, the two public goods play distinct roles:  $G_{1,t}$  enhances the productivity of capital while  $G_{2,t}$  augments labor efficiency. When the government collects a wealth tax and splits its revenue between these two goods, an efficient balance must be struck. If too much is devoted to  $G_{1,t}$  (i.e.,  $\mu$  is too high), the improvement in capital productivity comes at the expense of labor-enhancing services, which are crucial for mobilizing the labor input. Conversely, if  $\mu$  is too low, then insufficient resources are allocated to capital-enhancing infrastructure, and the marginal benefit of additional public spending declines. The first-order condition equates the proportional gain from a marginal shift in  $\mu$  in the capital channel with the corresponding loss in the labor channel. The unique solution  $\mu^* = \beta/(1-\alpha)$  thus balances these opposing effects and maximizes the overall marginal productivity of capital. Since the gross rental rate is proportional to this marginal productivity, it (and therefore the net return on capital, after subtracting the wealth tax) is maximized when  $\mu = \mu^*$ . This finding is significant because it demonstrates that, regardless of distributional heterogeneity in the economy, the growth-maximizing composition of public spending is determined solely by the production technology parameters  $\alpha$  and  $\beta$ , thereby providing a clear benchmark for efficient fiscal policy.

### Fiscal Policy Multipliers

The elasticities of factor prices to fiscal parameters quantify policy effectiveness:

$$\mathcal{E}_{r,\tau} = \frac{\partial \ln r}{\partial \ln \tau} = 1 - \alpha, \quad \mathcal{E}_{r,\mu} = \frac{\partial \ln r}{\partial \ln \mu} = \beta - \frac{\mu(1 - \alpha - \beta)}{1 - \mu},$$

$$\mathcal{E}_{\omega,\tau} = \frac{\partial \ln \omega}{\partial \ln \tau} = 1 - \alpha, \quad \mathcal{E}_{\omega,\mu} = \frac{\partial \ln \omega}{\partial \ln \mu} = \beta - \frac{\mu(1 - \alpha - \beta)}{1 - \mu}.$$

The symmetric  $\tau$ -elasticities ( $\mathcal{E}_{r,\tau} = \mathcal{E}_{\omega,\tau}$ ) reveal that taxation proportionally scales public goods provision, benefiting both factors. Although the  $\mu$ -elasticities are also equal, their distri-



butional implications differ. When  $\mu < \beta/(1-\alpha)$ , infrastructure spending increases both returns but tends to raise capital income more than labor income, due to capital's larger share  $\alpha > 1-\alpha-\beta$ . This highlights the political economy tension: policymakers must balance capital accumulation (via  $G_1$ ) with human capital development (via  $G_2$ ).

# **Production Approach**

To derive aggregate output via the production approach, we begin with the firm-level production function and systematically incorporate the government's role in providing productive public inputs. Substitute the public inputs  $G_{1,t} = \mu \tau_t K_t$  and  $G_{2,t} = (1 - \mu)\tau_t K_t$ , which are financed by tax revenues:

$$Y_{j,t} = AK_{j,t}^{\alpha} \left(\mu \tau_t K_t\right)^{\beta} \left[ (1-\mu)\tau_t K_t \cdot L_{j,t} \right]^{1-\alpha-\beta}$$

In symmetric equilibrium, all firms employ identical capital-labor ratios  $\frac{K_{j,t}}{L_{j,t}} = \frac{K_t}{L}$ . Aggregating across m firms by setting  $K_{j,t} = \frac{K_t}{m}$  and  $L_{j,t} = \frac{L}{m}$ , we sum outputs:

$$Y_t = \sum_{j=1}^{m} Y_{j,t} = m \cdot A \left(\frac{K_t}{m}\right)^{\alpha} \left(\mu \tau_t K_t\right)^{\beta} \left[ (1-\mu)\tau_t K_t \cdot \frac{L}{m} \right]^{1-\alpha-\beta}$$

Simplify exponents and regroup terms:

$$Y_t = A\mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} \tau_t^{1 - \alpha} L^{1 - \alpha - \beta} K_t$$

By normalizing m = 1, we consolidate firms into a representative unit without altering equilibrium dynamics.

## 3.4 Heterogeneous Population

The economy comprises a continuum of individuals differentiated by their initial endowments of labor and capital. Each individual i supplies labor inelastically, denoted  $l_i > 0$ , and holds a stock of capital  $k_{i,t} \ge 0$  at time t. Aggregate labor supply  $L = \int l_i di$  is constant, while aggregate capital  $K_t = \int k_{i,t} di$  evolves endogenously. Critically, individuals differ in their relative factor endowments, defined as the ratio of their labor-to-capital shares compared to the economy-wide average. Formally, individual i's relative factor endowment is:

$$\sigma_i \equiv \frac{l_i/L}{k_{i,t}/K_t} = \frac{l_i/k_{i,t}}{L/K_t}, \quad \sigma_i \ge 0.$$

This metric  $\sigma_i$  quantifies individual *i*'s economic dependence on labor income relative to capital income. A pure capitalist, with  $l_i = 0$ , has  $\sigma_i = 0$ , while a pure laborer, with  $k_{i,t} = 0$ , approaches  $\sigma_i \to \infty$ . The population median of  $\sigma_i$  typically exceeds unity in economies with right-skewed wealth distributions, reflecting the empirical regularity that labor income is more evenly distributed than capital ownership.

Under balanced growth, all individuals' capital stocks grow at the common rate  $\gamma(\tau, \mu) = r(\tau, \mu) - \tau - \rho$ , ensuring  $\sigma_i$  remains constant over time. This invariance arises because:

$$\frac{\dot{k}_{i,t}}{k_{i,t}} = \gamma(\tau,\mu) = \frac{\dot{K}_t}{K_t}, \quad \forall i,$$



so the ratio  $k_{i,t}/K_t$ —and hence  $\sigma_i$ —is time-independent.

The parameter  $\sigma_i$  governs individuals' conflicting interests over fiscal policy. Since laborenhancing public goods  $G_{2,t}$  raise wages proportionally to  $K_t$ , while capital-augmenting  $G_{1,t}$ elevates the return on capital  $r(\tau, \mu)$ , the allocation parameter  $\mu$  creates distributional tradeoffs. Individuals with  $\sigma_i > 1$  (labor-dependent) benefit disproportionately from  $G_{2,t}$ , as their income relies more on wages. Conversely, those with  $\sigma_i < 1$  (capital-rich) favor  $G_{1,t}$  to amplify capital returns. These dynamics generate political conflict over  $\mu$ , with the equilibrium allocation reflecting the preferences of the pivotal voter.

The tax rate  $\tau$  adjusts endogenously to satisfy the government budget constraint  $G_{1,t} + G_{2,t} =$  $\tau K_t$ . A higher  $\mu$  increases  $G_{1,t}$  but necessitates a higher  $\tau$  to maintain  $G_{2,t}$ , ceteris paribus. This dual role of  $\mu$ —shaping both public goods composition and the tax burden—links distributional preferences to growth outcomes. Labor-dependent individuals  $(\sigma_i > 1)$  thus face a trilemma: advocate for lower  $\mu$  to boost  $G_{2,t}$ , accept higher  $\tau$  to fund it, or tolerate slower wage growth. The resolution of this tension hinges on the elasticity of wages and interest rates to  $\mu$ , which we derive in the next subsection.

#### 3.5 Economic Behavior of Individual i

Each individual i inelastically supplies labor  $l_i$  and accumulates capital  $k_{i,t}$ , maximizing intertemporal utility over an infinite horizon. The population is heterogeneous in initial endowments  $(k_{i,0}, l_i)$ , with aggregate capital  $K_t = \int k_{i,t} di$  and labor  $L = \int l_i di$ . Preferences are logarithmic with discount rate  $\rho > 0$ :

$$U_i = \int_0^\infty \ln c_{i,t} \, e^{-\rho t} dt,$$

subject to the wealth accumulation equation:

$$\dot{k}_{i,t} = \underbrace{\left[r(\tau,\mu) - \tau\right] k_{i,t}}_{\text{Net capital income}} + \underbrace{\omega(\tau,\mu) K_t l_i}_{\text{Labor income}} - c_{i,t},$$

where  $r(\tau, \mu)$  and  $\omega(\tau, \mu)$  are equilibrium factor prices from Section 3.3, and the No-Ponzi condition:

$$\lim_{t\to\infty} k_{i,t} \exp\left(-\int_0^t \left[r(\tau,\mu) - \tau\right] ds\right) \ge 0.$$

where  $\lambda_{i,t}$  is the costate variable. First-order conditions yielding the Keynes-Ramsey rule:

$$\frac{\dot{c}_{i,t}}{c_{i,t}} = \underbrace{\left[r(\tau,\mu) - \tau\right] - \rho}_{\gamma(\tau,\mu)}.$$

Consumption grows at the constant rate  $\gamma(\tau,\mu)$ , determined by the net return on capital. In symmetric equilibrium, all capital grows at rate  $\gamma(\tau,\mu)$ :

$$\frac{\dot{k}_{i,t}}{k_{i,t}} = \frac{\dot{K}_t}{K_t} = \gamma(\tau, \mu).$$

The long-run solvency constraint ensures:

$$\lim_{t \to \infty} \lambda_{i,t} k_{i,t} e^{-\rho t} = \lim_{t \to \infty} \frac{k_{i,t}}{c_{i,t}} e^{-\rho t} = 0.$$



Substituting  $c_{i,t} = c_{i,0}e^{\gamma t}$  and  $k_{i,t} = k_{i,0}e^{\gamma t}$ :

$$\lim_{t \to \infty} \frac{k_{i,0}}{c_{i,0}} e^{-(\rho - \gamma)t} = 0 \quad \text{if} \quad \rho > \gamma(\tau, \mu).$$

### 3.6 Transversality and Viability Constraints

The equilibrium growth path must satisfy two fundamental constraints for economic viability:

1. **Positive Growth Condition**: The net return on capital must sufficiently compensate for intertemporal discounting:

$$\gamma(\tau,\mu) = \underbrace{\left[r(\tau,\mu) - \tau\right]}_{\text{Net capital return}} -\rho > 0 \implies r(\tau,\mu) - \tau > \rho.$$

This ensures capital accumulation outpaces consumption discounting, avoiding stagnation.

2. **Transversality Condition**: The No-Ponzi constraint imposes an upper bound on sustainable growth:

$$\rho > \gamma(\tau, \mu) \quad \Rightarrow \quad r(\tau, \mu) - \tau < 2\rho.$$

This prevents explosive paths where capital grows faster than discounted utility valuations. Combining these, fiscal policy  $(\tau, \mu)$  must satisfy:

$$\rho < r(\tau, \mu) - \tau < 2\rho.$$

Substituting  $r(\tau, \mu) = \alpha A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} \tau^{1 - \alpha} L^{1 - \alpha - \beta}$ , the viability corridor becomes:

$$\rho < \alpha A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} \tau^{1 - \alpha} L^{1 - \alpha - \beta} - \tau < 2\rho.$$

This dual inequality defines the *fiscal possibility frontier* - tax rates and public goods allocations that balance productivity gains from infrastructure/human capital against the distortions of wealth taxation. This corridor defines the set of feasible tax rates and public goods allocations that balance productivity gains from infrastructure and human capital against the distortions of wealth taxation.

Let  $r(\tau,\mu) = \alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{1-\alpha} L^{1-\alpha-\beta}$  and define  $F(\tau;\mu) = r(\tau,\mu) - \tau$ . Fix parameters  $0 < \alpha < 1, \ 0 < \beta < 1, \ \alpha + \beta < 1, \ \text{and} \ \rho > 0$ . For each  $\mu \in (0,1)$  we have: (1).  $F(\cdot;\mu)$  is continuous, strictly concave on  $\tau > 0$ , with  $\lim_{\tau \to 0^+} F(\tau;\mu) = 0$  and  $\lim_{\tau \to \infty} F(\tau;\mu) = -\infty$ . (2). Its unique interior maximizer  $\tau^*(\mu)$  is

$$\tau^*(\mu) = \left[ (1 - \alpha) \alpha A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} L^{1 - \alpha - \beta} \right]^{1/\alpha}.$$

Assume moreover the viability condition  $F(\tau^*(\mu); \mu) > 2\rho$ . Then, there exists a unique  $\tau_{\min}(\mu) \in (0, \tau^*(\mu))$  such that  $F(\tau_{\min}(\mu); \mu) = \rho$ . and there exists a unique  $\tau_{\max}(\mu) \in (\tau^*(\mu), \infty)$  such that  $F(\tau_{\max}(\mu); \mu) = 2\rho$ . Hence the set of tax rates that deliver balanced growth,  $\rho < F(\tau; \mu) < 2\rho$ , is exactly the open interval

$$\{\tau > 0: \ \tau_{\min}(\mu) < \tau < \tau_{\max}(\mu)\}.$$



Moreover, defining  $\Psi(\mu) = \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta}$ , one shows (as before) that  $\Psi$  is maximized at  $\mu^* = 0$  $\beta/(1-\alpha)$ , and therefore both  $\tau_{\min}(\mu)$  and  $\tau_{\max}(\mu)$  are increasing in  $\mu$  for  $\mu < \mu^*$  and decreasing for  $\mu > \mu^*$ .

The above result implies that in order for the economy to sustain a balanced growth path—where the net return on capital lies within the interval  $(\rho, 2\rho)$ —the wealth tax rate must be chosen within a viability corridor whose endpoints depend on the composition of public spending. In particular, the highest possible range of feasible tax rates is attained when the government allocates its revenues in the proportion  $\mu^* = \beta/(1-\alpha)$ , which optimally balances the contributions of capital-augmenting and labor-augmenting public goods. Deviations from this optimal allocation (either favoring too much infrastructure or too much human-capital enhancement) narrow the set of tax rates that can sustain balanced growth, thereby highlighting the importance of matching fiscal composition to the underlying production technology.

#### 3.7 Comparative Statics of Fiscal Policy

The growth rate  $\gamma(\tau,\mu)$  responds non-monotonically to changes in  $\tau$  and  $\mu$ , reflecting the dual role of fiscal policy in providing public goods and distorting capital accumulation.

Tax Rate Elasticity: Differentiating  $\gamma$  with respect to  $\tau$ :

$$\frac{\partial \gamma}{\partial \tau} = \underbrace{(1-\alpha)\alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{-\alpha} L^{1-\alpha-\beta}}_{Productivity \ Effect} - \underbrace{1}_{Tax \ Distortion}.$$

The first term captures the productivity gains from higher public goods provision, while the second term represents the direct burden of taxation. The critical tax rate  $\tau^*(\mu)$  that maximizes  $\gamma$  satisfies:

$$\tau^*(\mu) = \left[\alpha(1-\alpha)A\mu^{\beta}(1-\mu)^{1-\alpha-\beta}L^{1-\alpha-\beta}\right]^{1/\alpha}.$$

For  $\tau < \tau^*(\mu)$ , the productivity effect dominates, and raising  $\tau$  accelerates growth. Beyond  $\tau^*(\mu)$ , excessive taxation stifles capital accumulation, reducing growth.

The Figure 2 illustrates the non-monotonic relationship between the balanced-growth rate  $\gamma(\tau)$  and the wealth tax rate  $\tau$ . With the chosen parameter values, the function

$$\gamma(\tau) = 0.9963 \,\tau^{0.7} - \tau - 0.05$$

first increases as  $\tau$  rises from zero, reflecting that moderate increases in the tax rate boost public goods provision (the "Productivity Effect"). This positive effect dominates for  $\tau < \tau^* \approx 0.30$ , where the economy experiences accelerated growth. However, beyond this critical tax rate the tax distortion becomes too severe, and further increases in  $\tau$  reduce the net return on capital, thereby lowering  $\gamma(\tau)$ . This graph thereby provides a clear visual representation of the trade-off underlying fiscal policy in the model.

Public Goods Allocation Elasticity: Differentiating  $\gamma$  with respect to  $\mu$ :

$$\frac{\partial \gamma}{\partial \mu} = \alpha A \tau^{1-\alpha} L^{1-\alpha-\beta} \mu^{\beta-1} (1-\mu)^{-\alpha-\beta} \left[ \beta (1-\mu) - (1-\alpha-\beta) \mu \right].$$



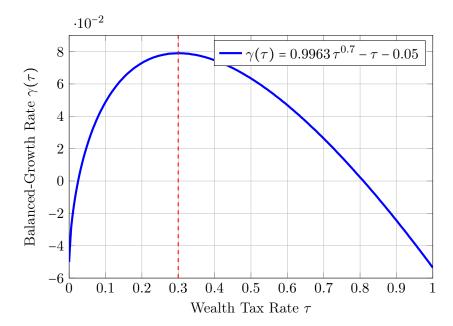


Figure 2. Relationship between the balanced-growth rate  $\gamma(\tau)$  and the wealth tax rate  $\tau$ , for fixed parameter values  $\alpha=0.3,\ \beta=0.2,\ A=5.06,\ L=1,\ \mu=0.25,\ {\rm and}\ \rho=0.05.$  The function  $\gamma(\tau)=0.9963\ \tau^{0.7}-\tau-0.05$  attains its maximum at  $\tau^*\approx 0.30$ . For  $\tau<\tau^*$ , the productivity gains from increased public goods provision dominate the distortionary effect of taxation, so that  $\gamma(\tau)$  rises with  $\tau$ . Conversely, for  $\tau>\tau^*$  the distortionary burden of higher taxation outweighs its benefits, causing  $\gamma(\tau)$  to decline.

The growth-maximizing allocation  $\mu^*$  balances the marginal products of  $G_1$  and  $G_2$ :

$$\mu^* = \frac{\beta}{1 - \alpha}.$$

When  $\mu < \mu^*$ , reallocating funds toward capital-augmenting  $G_1$  raises growth by enhancing capital productivity. Conversely, when  $\mu > \mu^*$ , excessive  $G_1$  crowds out labor-enhancing  $G_2$ , depressing aggregate efficiency.

The extended model generalizes the Alesina-Rodrik framework by introducing a policy tradeoff between two types of public goods. Infrastructure  $(G_1)$  and human capital  $(G_2)$  exhibit competing marginal returns. Infrastructure-intensive policies  $(\mu \uparrow)$  elevate capital's marginal product, benefiting wealthier households reliant on capital income while human capital-intensive policies  $(\mu \downarrow)$  raise labor productivity, favoring workers dependent on wage income.

The growth-maximizing fiscal policy  $(\tau^*, \mu^*)$  internalizes these trade-offs,  $\tau^*$  ensures public goods provision optimally offsets tax distortions while  $\mu^*$  balances the productivity contributions of  $G_1$  and  $G_2$ , avoiding overinvestment in either sector. Deviations from  $(\tau^*, \mu^*)$  reduce growth, creating a political economy tension: voters' preferred policies depend on their relative factor endowments, as analyzed in the subsequent section.

The figure 3 illustrates the non-monotonic relationship between the balanced-growth rate  $\gamma(\tau,\mu)$  and the public goods allocation parameter  $\mu$ . In this specification, with fixed values  $\alpha = 0.3$ ,  $\beta = 0.2$ , A = 5.06,  $\tau = 0.2$ , L = 1, and  $\rho = 0.05$ , the function

$$\gamma(\tau,\mu) = 0.4932 \,\mu^{0.2} (1-\mu)^{0.5} - 0.25$$



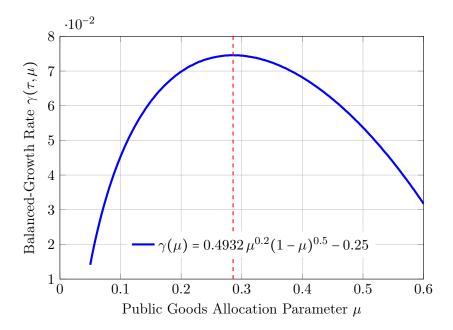


Figure 3. Relationship between the balanced-growth rate  $\gamma(\tau,\mu)$  and the public goods allocation parameter  $\mu$ , for fixed parameter values  $\alpha = 0.3$ ,  $\beta = 0.2$ , A = 5.06, L = 1,  $\tau = 0.2$ , and  $\rho = 0.05$ . The function  $\gamma(\tau, \mu) = 0.4932 \,\mu^{0.2} (1 - \mu)^{0.5} - 0.25$  attains its maximum at  $\mu^* = \beta/(1 - \alpha) \approx 0.29$ , where  $\gamma(\tau, \mu^*) \approx 0.075$ . For  $\mu < \mu^*$ , shifting more resources toward capital-augmenting public goods raises the balanced-growth rate; for  $\mu > \mu^*$ , the crowding-out effect on labor-enhancing services dominates and  $\gamma(\tau,\mu)$  declines.

captures how reallocating tax revenue between capital-augmenting public goods  $(G_1)$  and laborenhancing public goods  $(G_2)$  affects the net return on capital and thus economic growth. The curve reaches its maximum at the growth-maximizing allocation  $\mu^* \approx 0.29$ . For values of  $\mu$  below this optimum, increasing  $\mu$  (i.e., shifting spending toward  $G_1$ ) enhances capital productivity and boosts growth. However, for  $\mu$  above  $\mu^*$ , the negative effect of crowding out labor-enhancing services becomes dominant, causing the balanced-growth rate to decline. This trade-off is central to the model' implications for fiscal policy.

In the extended Alesina-Rodrik framework described above, suppose that fiscal policy is characterized by the pair  $(\tau, \mu)$ , where  $\tau$  is the constant wealth tax rate and  $\mu$  is the share of tax revenue devoted to capital-augmenting public goods. Then there exists a unique second-best fiscal policy pair  $(\tau^*, \mu^*)$  that maximizes the balanced-growth rate

$$\gamma(\tau,\mu) = r(\tau,\mu) - \tau - \rho,$$

where

$$r(\tau,\mu) = \alpha A \,\mu^{\beta} (1-\mu)^{1-\alpha-\beta} \,\tau^{1-\alpha} L^{1-\alpha-\beta}.$$

Specifically, the unique optimum satisfies

$$\mu^* = \frac{\beta}{1 - \alpha}$$

and  $\tau^*$  is the unique solution to

$$\alpha A (1-\alpha) L^{1-\alpha} \, \mu^\beta (1-\mu)^{\, 1-\alpha-\beta} \, \tau^{-\alpha} = 1, \label{eq:alpha-def}$$



so that  $\tau^* = [\alpha A(1-\alpha)L^{1-\alpha}\mu^{\beta}(1-\mu)^{1-\alpha-\beta}]^{1/\alpha}$ . Moreover, for any feasible fiscal policy  $(\tau,\mu)$  different from  $(\tau^*,\mu^*)$  the balanced–growth rate is strictly lower than  $\gamma(\tau^*,\mu^*)$ , and, near the optimum, the loss in  $\gamma$  is approximately quadratic in the deviation of  $\tau$  from  $\tau^*$ .

The proposition establishes that there is a unique fiscal policy—defined by the tax rate  $\tau^*$  and the public spending composition  $\mu^*$ —that maximizes the economy's balanced–growth rate. The optimal composition  $\mu^*$  is determined solely by the technological parameters of the production function and equals  $\beta/(1-\alpha)$ ; this ratio represents the precise balance between capital-augmenting and labor-augmenting public spending that maximizes the marginal productivity of capital. Moreover, given any deviation of the tax rate from its optimal level  $\tau^*$  (holding the spending composition at its optimum), the resulting decline in the growth rate is convex in the deviation, implying that fiscal policy mistakes yield disproportionately large growth losses. The economic intuition is that, when the government finances public goods solely through a wealth tax, there is an inherent trade-off: higher taxation increases the stock of productive public inputs and thereby enhances the productivity of private capital, but it also directly reduces the net return on capital and distorts saving behavior. The optimal policy strikes a delicate balance between these opposing effects. Deviating from this second–best policy—either by misallocating public spending or by setting the tax rate too high or too low—leads to suboptimal growth outcomes, and the loss from such deviations increases quadratically in magnitude.

## 4. Voters' Indirect Utility and Optimal Policy

In this section we derive, step by step and in full detail, the indirect utility function for an individual voter in our extended Alesina–Rodrik framework with dual public goods. In our model each individual i has infinite-horizon logarithmic preferences given by

$$U_i = \int_0^\infty \ln c_{i,t} \, e^{-\rho t} \, dt,$$

where  $\rho > 0$  denotes the time-preference rate. Under balanced growth the consumption path for individual i takes the form  $c_{i,t} = c_{i,0} e^{\gamma(\tau,\mu)t}$ , with  $\gamma(\tau,\mu)$  representing the common growth rate, which in equilibrium is determined by  $\gamma(\tau,\mu) = r(\tau,\mu) - \tau - \rho$ , where  $r(\tau,\mu)$  is the gross return on capital and  $\tau$  is the wealth tax. In our framework the initial consumption is determined by the steady-state relation

$$c_{i,0} = \left[\omega(\tau,\mu)L\,\sigma_i + \rho\right]k_{i,0},$$

where  $\omega(\tau,\mu)$  is the per-unit wage (with aggregate wage given by  $w_t = \omega(\tau,\mu)K_t$ ), L is aggregate labor,  $k_{i,0}$  is individual i's initial capital, and  $\sigma_i \equiv \frac{l_i/k_{i,0}}{L/K_0}$  measures the relative dependence of individual i on labor income relative to capital income. Substituting the consumption path into the utility integral we obtain

$$U_i = \int_0^{\infty} \left[ \ln c_{i,0} + \gamma(\tau, \mu) t \right] e^{-\rho t} dt.$$

Because

$$\int_0^\infty e^{-\rho t}\,dt = \frac{1}{\rho} \quad \text{and} \quad \int_0^\infty t\,e^{-\rho t}\,dt = \frac{1}{\rho^2},$$



we deduce that

$$U_i = \frac{1}{\rho} \ln c_{i,0} + \frac{\gamma(\tau, \mu)}{\rho^2}.$$

Replacing  $c_{i,0}$  by its expression, we obtain the full indirect utility function

$$U_i(\tau,\mu) = \frac{1}{\rho} \ln \left( \omega(\tau,\mu) L \sigma_i + \rho \right) + \frac{1}{\rho} \ln k_{i,0} + \frac{r(\tau,\mu) - \tau - \rho}{\rho^2}.$$

Since the term  $\frac{1}{\rho} \ln k_{i,0}$  is independent of the policy parameters, the relevant part for policy choice is given by

$$\widetilde{U}_i(\tau,\mu) = \frac{1}{\rho} \ln \left( \omega(\tau,\mu) L \, \sigma_i + \rho \right) + \frac{r(\tau,\mu) - \tau - \rho}{\rho^2} \, .$$

The function  $\widetilde{U}_i(\tau,\mu)$  captures both the level effect on consumption via the wage channel and the growth effect through the net return on capital. The term inside the logarithm,  $\omega(\tau,\mu)L\sigma_i + \rho$ , reflects the initial consumption level, which depends on the wage rate, the aggregate labor force, and the individual's relative endowment  $\sigma_i$ , while the term  $(r(\tau, \mu) - \tau - \rho)$  represents the long-run growth rate of consumption.

#### 4.1 Relationship between $\tau_i$ and $\sigma_i$

To show that an individual voter's preferred tax rate  $\tau_i$  increases with her relative factor endowment  $\sigma_i$ , we begin with the first-order condition (FOC) for the optimal tax rate derived from the indirect utility function.

$$\widetilde{U}_i(\tau,\mu) = \frac{1}{\rho} \ln \left( \omega(\tau,\mu) L \, \sigma_i + \rho \right) + \frac{r(\tau,\mu) - \tau - \rho}{\rho^2} \,,$$

so that the FOC with respect to  $\tau$  (holding  $\mu$  fixed) is

$$\frac{1}{\rho} \frac{\partial}{\partial \tau} \ln \left[ \omega(\tau, \mu) L \, \sigma_i + \rho \right] + \frac{1}{\rho^2} \left[ r_\tau(\tau, \mu) - 1 \right] = 0. \tag{3}$$

Here,  $r_{\tau}(\tau, \mu)$  and  $\omega_{\tau}(\tau, \mu)$  denote the partial derivatives of  $r(\tau, \mu)$  and  $\omega(\tau, \mu)$  with respect to  $\tau$ . Equation (3) implicitly defines the individual's optimal tax rate  $\tau_i$  as a function of her relative endowment  $\sigma_i$ , i.e.,  $\tau_i = \tau(\sigma_i)$ .

To determine the effect of a change in  $\sigma_i$  on  $\tau_i$ , we totally differentiate the FOC in (3) with respect to  $\sigma_i$ . Denote by

$$F(\tau, \sigma_i) = \frac{1}{\rho} \frac{\omega_{\tau}(\tau, \mu) L \sigma_i}{\omega(\tau, \mu) L \sigma_i + \rho} + \frac{1}{\rho^2} \left[ r_{\tau}(\tau, \mu) - 1 \right],$$

so that the optimum satisfies

$$F(\tau, \sigma_i) = 0$$
.

By the implicit function theorem, we have

$$\frac{d\tau}{d\sigma_i} = -\frac{\partial F/\partial \sigma_i}{\partial F/\partial \tau} \,. \tag{4}$$

As shown in Appendix A.3,

$$\frac{\partial F}{\partial \sigma_i} > 0$$
 and  $\frac{\partial F}{\partial \tau} < 0$ ,



it follows that

$$\frac{d\tau}{d\sigma_i} = -\frac{(+)}{(-)} > 0.$$

This positive derivative implies that individuals who are more dependent on labor income (i.e., with higher  $\sigma_i$ ) have a higher preferred tax rate. The intuition is that, since labor income is less distorted by the wealth tax (which subtracts a fixed amount from capital income), these individuals benefit more from increased public spending and are willing to tolerate a higher tax rate. In contrast, individuals who rely more on capital income (with lower  $\sigma_i$ ) are adversely affected by the tax and therefore prefer a lower tax rate.

# 4.2 Relationship between $\mu$ and $\sigma_i$

To assess how voters' relative factor endowments affect their preferred allocation of public goods, we derive the optimal value of  $\mu$  by differentiating the indirect utility function  $\widetilde{U}_i$  with respect to  $\mu$ . In particular, we have

$$\frac{\partial \widetilde{U}_{i}}{\partial \mu} = \underbrace{\frac{1}{\rho} \frac{\omega_{\mu} L \sigma_{i}}{\omega L \sigma_{i} + \rho}}_{\text{Marginal wage effect}} + \underbrace{\frac{1}{\rho^{2}} r_{\mu}}_{\text{Marginal growth effect}} = 0,$$

where  $\omega_{\mu} \equiv \partial \omega / \partial \mu$  and  $r_{\mu} \equiv \partial r / \partial \mu$ . Using the equilibrium expressions for wages and interest rates:

$$\omega(\tau,\mu) = (1 - \alpha - \beta)A\mu^{\beta}(1 - \mu)^{1-\alpha-\beta}\tau^{1-\alpha}L^{-(\alpha+\beta)},$$
  
$$r(\tau,\mu) = \alpha A\mu^{\beta}(1 - \mu)^{1-\alpha-\beta}\tau^{1-\alpha}L^{1-\alpha-\beta}.$$

Differentiate  $\omega(\tau, \mu)$  with respect to  $\mu$ :

$$\omega_{\mu} = \omega \left( \frac{\beta}{\mu} - \frac{1 - \alpha - \beta}{1 - \mu} \right).$$

Similarly, differentiate  $r(\tau, \mu)$  with respect to  $\mu$ :

$$r_{\mu} = r \cdot \left(\frac{\beta}{\mu} - \frac{1 - \alpha - \beta}{1 - \mu}\right).$$

Insert  $\omega_{\mu}$  and  $r_{\mu}$  into the first-order condition and factor out the common term  $\left(\frac{\beta}{\mu} - \frac{1-\alpha-\beta}{1-\mu}\right)$ 

$$\left(\frac{\beta}{\mu} - \frac{1 - \alpha - \beta}{1 - \mu}\right) \left[\frac{\omega L \sigma_i}{\rho(\omega L \sigma_i + \rho)} + \frac{r}{\rho^2}\right] = 0.$$

The term in brackets is strictly positive:

$$\frac{\omega L \sigma_i}{\rho(\omega L \sigma_i + \rho)} + \frac{r}{\rho^2} > 0 \quad \text{(since } \omega, r, \rho > 0\text{)}.$$

Thus, the non-trivial solution requires:

$$\frac{\beta}{\mu} - \frac{1 - \alpha - \beta}{1 - \mu} = 0.$$



Solve for  $\mu$ :

$$\mu^* = \frac{\beta}{1 - \alpha}.$$

This solution depends only on technology parameters  $(\alpha, \beta)$ , not on individual endowments  $(\sigma_i)$ . Consequently:

$$\frac{\partial \mu^*}{\partial \sigma_i} = 0.$$

The independence of the growth-maximizing public goods allocation ( $\mu^* = \beta/(1-\alpha)$ ) from individual endowments reflects a fundamental tension between productive efficiency and distributional politics. While political preferences over  $\mu$  naturally diverge across individuals depending on their relative reliance on labor versus capital income  $(\sigma_i)$ , the optimal composition of public expenditure is determined purely by technological considerations.

This result arises because  $\mu^*$  balances the marginal productivity contributions of infrastructure  $(G_1)$  and labor-enhancing services  $(G_2)$  in the aggregate production function. Infrastructure amplifies capital's productivity through  $\beta$ , while labor services elevate wages through  $(1-\alpha-\beta)$ . The Cobb-Douglas technology forces these elasticities to sum to  $1-\alpha$ , creating a fixed trade-off: each percentage point reallocated from  $G_2$  to  $G_1$  raises capital's marginal product by  $\beta/(1-\alpha-\beta)$ times the percentage loss in labor's marginal product. The unique solution  $\mu^* = \beta/(1-\alpha)$  equalizes these competing marginal effects, maximizing the economy-wide return on capital and thus long-run growth.

Crucially, this efficiency condition depends only on deep parameters of the production technology  $(\alpha, \beta)$ , not on the distribution of factor ownership  $(\sigma_i)$ . Even in highly unequal societies where median voters favor labor-intensive public spending  $(\mu < \mu^*)$ , or capital-dominated polities pushing for infrastructure overinvestment ( $\mu > \mu^*$ ), the growth-optimal allocation remains unchanged. The disconnect between political preferences and productive efficiency creates a latent conflict: while democratic processes tend to align  $\mu$  with the median voter's  $\sigma_i$ , such allocations generally deviate from  $\mu^*$ , depressing growth below its technological potential. This tension underscores the model's central insight - that growth-maximizing fiscal policy requires insulating expenditure composition from short-term distributive pressures, a challenge exacerbated by the inverse relationship between inequality and growth in political equilibrium.

#### **5.** Distributional Conflict, Fiscal Policy, and Growth

The interplay between inequality, fiscal policy, and economic growth emerges as a central tension in political economies with heterogeneous endowments. Our framework demonstrates how the distribution of factor ownership shapes policy choices, which in turn determine long-run growth trajectories. This section synthesizes two key mechanisms: (i) how inequality induces suboptimal taxation that depresses growth, and (ii) the conditions under which such dynamics culminate in growth traps.



### 5.1 The Redistribution-Growth Trade-off

The model's political economy equilibrium reveals a fundamental trade-off: societies with greater wealth inequality tend to adopt tax policies that prioritize redistribution over growth. This arises because the median voter's relative factor endowment  $\sigma^m$ —measuring their dependence on labor versus capital income—determines the preferred tax rate  $\tau^m$ . As shown in Proposition ??, when  $\sigma^m > 1$  (reflecting a median voter more reliant on wages than capital returns), the politically chosen tax rate  $\tau^m$  exceeds the growth-maximizing rate  $\tau^*$ . The resulting fiscal policy tilts resources toward immediate consumption and redistribution at the expense of capital accumulation, slowing long-run growth.

The intuition is twofold. First, labor income, being more evenly distributed than capital ownership, constitutes a larger share of total income for the median voter. This voter thus benefits disproportionately from public goods that enhance wages, even if financed by growth-inhibiting taxes on capital. Second, the concavity of the growth function  $\gamma(\tau)$  ensures that deviations from  $\tau^*$  impose quadratic losses on growth—moderate inequality induces modest growth reductions, while severe inequality triggers precipitous declines.

Consider the extended Alesina–Rodrik framework in which each individual's indirect utility is given by

$$\widetilde{U}_i(\tau,\mu) = \frac{1}{\rho} \ln \left( w(\tau,\mu) L \, \sigma_i + \rho \right) + \frac{r(\tau,\mu) - \tau - \rho}{\rho^2},$$

with the equilibrium gross rental rate

$$r(\tau,\mu) = \alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{1-\alpha} L^{1-\alpha-\beta},$$

and where the government finances two public goods via a wealth tax at rate  $\tau$ , allocated according to a parameter  $\mu$ . Suppose that (i) an individual's preferred tax rate is strictly increasing in her relative factor endowment  $\sigma_i$  (that is,  $\frac{\partial \tau_i}{\partial \sigma_i} > 0$ ), and (ii) the growth rate on the balanced-growth path is given by

$$\gamma(\tau,\mu) = r(\tau,\mu) - \tau - \rho,$$

which is strictly concave in  $\tau$  with maximum at  $\tau = \tau^*$  corresponding to a representative agent with  $\sigma = 1$ , and with the optimal public goods composition fixed at  $\mu^* = \frac{\beta}{1-\alpha}$ . Then, if the median voter's relative factor endowment satisfies  $\sigma^m > 1$ , the tax rate chosen by majority voting,  $\tau^m$ , will exceed the growth-maximizing tax rate  $\tau^*$ , and the corresponding balanced-growth rate  $\gamma(\tau^m, \mu^*)$  will be lower than the maximum possible rate. In other words, an increase in the median relative factor endowment (reflecting higher inequality in favor of labor) leads to a higher politically chosen tax rate and, as a consequence of the concavity of  $\gamma(\tau, \mu^*)$ , to a lower long-run growth rate.

The economic intuition behind this result is as follows. In our framework, individuals who are more dependent on labor income (reflected by a higher  $\sigma_i$ ) are less adversely affected by a wealth tax—since capital income is taxed, a higher reliance on labor income makes one less vulnerable to the tax's distortions. Consequently, such individuals favor a higher tax rate. When the median voter exhibits a relatively high  $\sigma^m$ , the political process leads to the selection of a tax rate  $\tau^m$  that exceeds the level that would maximize growth if the representative agent were neutral (i.e.



with  $\sigma = 1$ ). Because the balanced-growth rate  $\gamma(\tau, \mu^*)$  is maximized at the tax rate  $\tau^*$ , any increase in  $\tau$  beyond  $\tau^*$  diminishes growth. Hence, a society with greater inequality (in the sense of a higher median relative dependence on labor income) will choose fiscal policies that, while redistributive, impose excessive distortions on capital accumulation, leading to a lower long-run per capita growth rate.

#### 5.2From Inequality to Growth Traps

The growth consequences of distributionally motivated taxation can become self-reinforcing. Proposition 5.2 identifies a critical threshold  $\bar{\sigma}$  beyond which inequality pushes the economy into a stagnation trap. When the median voter's relative labor dependence  $\sigma^m$  surpasses  $\bar{\sigma}$ , the equilibrium tax rate  $\tau^m$  becomes so high that the net return on capital falls below the time preference rate  $(r(\tau^m) - \tau^m < \rho)$ . This violates the viability condition for positive growth, causing capital stock and output to contract indefinitely.

Such traps emerge from the interplay of political and economic dynamics. High inequality increases the median voter's preferred tax rate, which reduces capital accumulation and lowers wages over time. Paradoxically, the wage decline amplifies the median voter's relative reliance on labor income ( $\sigma^m$  rises), further increasing political pressure for redistribution. This feedback loop entrenches suboptimal fiscal policies, locking the economy into a low-growth equilibrium.

Consider the extended Alesina–Rodrik framework in which aggregate output is given by

$$Y_t = A K_t^{\alpha} \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} \tau^{1 - \alpha} L^{1 - \alpha - \beta},$$

and the balanced-growth rate is determined by

$$\gamma(\tau,\mu) = r(\tau,\mu) - \tau - \rho,$$

with the gross rental rate of capital

$$r(\tau,\mu) = \alpha A \,\mu^{\beta} (1-\mu)^{1-\alpha-\beta} \,\tau^{1-\alpha} \,L^{1-\alpha-\beta}.$$

Suppose that the tax rate preferred by an individual with relative factor endowment  $\sigma$  is given by a function  $\tau(\sigma)$  that is strictly increasing in  $\sigma$ , with  $\tau(1) = \tau^*$  corresponding to the growthmaximizing tax rate. If the median voter in the population has relative factor endowment  $\sigma^m > 1$ , then the politically determined tax rate is  $\tau^m = \tau(\sigma^m) > \tau^*$ . Define  $\tau^z$  as the unique tax rate satisfying

$$r(\tau^z, \mu) - \tau^z = \rho,$$

and let  $\bar{\sigma} > 1$  be the unique value for which  $\tau(\bar{\sigma}) = \tau^z$ . Then, if  $\sigma^m > \bar{\sigma}$  the equilibrium tax rate satisfies

$$r(\tau^m, \mu) - \tau^m < \rho$$

so that the net return on capital is below the discount rate and balanced growth is unsustainable, whereas if  $\sigma^m < \bar{\sigma}$  then  $r(\tau^m, \mu) - \tau^m > \rho$ , ensuring positive growth.

The political determination of the tax rate is intimately linked to the distribution of factor endowments in society. In particular, because individual preferences lead to higher tax rates



when one relies more on labor income (that is, when  $\sigma > 1$ ), the median voter's relative factor endowment crucially influences the equilibrium tax rate. When the median voter is sufficiently labor-dependent—specifically, when  $\sigma^m$  exceeds a critical threshold  $\bar{\sigma}$ —the politically chosen tax rate becomes so high that the net return on capital drops below the rate of time preference. In this situation, the economy's balanced-growth rate turns negative, indicating that excessive redistribution (as reflected in an over-aggressive tax rate) can undermine long-run growth. This finding highlights a potential political economy trap whereby high levels of inequality (in the sense of a median voter with a high relative dependence on labor income) may lead to fiscally driven policies that, while redistributive, ultimately impede capital accumulation and economic growth.

The model underscores that growth-maximizing fiscal policy requires insulating expenditure composition from short-term distributive pressures. While the optimal allocation  $\mu^* = \beta/(1-\alpha)$  maximizes productive efficiency, democratic processes tend to align  $\mu$  with the median voter's  $\sigma^m$ , often deviating from  $\mu^*$ . Institutional safeguards—such as independent fiscal councils or constitutional spending rules—could mitigate this tension by binding policymakers to technocratic efficiency criteria.

However, the feasibility of such arrangements depends on the initial distribution of endowments. In highly unequal societies, the growth trap threshold  $\bar{\sigma}$  may lie within the politically feasible range, leaving piecemeal reforms ineffective. Addressing such cases requires redistributive policies that directly alter the factor ownership distribution (e.g., wealth taxes, land reform), thereby shifting the median voter's preferences toward growth-compatible taxation. Our analysis thus identifies a dual role for policy: optimizing public goods provision given existing inequality, while gradually reshaping endowments to align political incentives with long-run growth.

The magnitude of the growth penalty from distributionally motivated taxation depends crucially on the *degree* of inequality in factor endowments. To formalize this relationship, we model the distribution of relative factor endowments  $\sigma_i$  as lognormal—a common characterization of wealth and income distributions. This allows us to derive the closed-form relationship between inequality, measured by the dispersion parameter  $\theta$ , and the deviation of the politically chosen tax rate from its growth-maximizing level.

Assume that in the extended Alesina–Rodrik framework the individual's preferred wealth tax rate is given by a continuously differentiable and strictly increasing function  $\tau(\sigma)$  of the relative factor endowment  $\sigma$ , with  $\tau(1) = \tau^*$  corresponding to the growth-maximizing tax rate. Suppose further that the distribution of  $\sigma$  is lognormal with  $\ln \sigma \sim N(-\theta/2, \theta)$  so that the geometric mean equals one and the median is  $\sigma^m = \exp(-\theta/2)$ , where  $\theta > 0$  measures dispersion. Then, to a first-order approximation, the percentage deviation of the median voter's preferred tax rate  $\tau^m \equiv \tau(\sigma^m)$  from the growth-maximizing tax rate  $\tau^*$  is given by

$$\frac{\tau^* - \tau^m}{\tau^*} \approx \frac{\tau'(1)}{\tau^*} \Big[ 1 - \exp\left(-\frac{\theta}{2}\right) \Big],$$

which is strictly increasing in  $\theta$  and equals zero when  $\theta = 0$ .

If individuals' optimal wealth tax rates increase with their relative reliance on labor income, then a higher dispersion in the distribution of relative factor endowments (as measured



by the variance parameter  $\theta$  of the lognormal distribution) leads to a median voter preference that deviates further from the growth-maximizing tax rate. In our setting the representative (or growth-maximizing) agent has  $\sigma = 1$ , so that  $\tau(1) = \tau^*$ . However, with a more unequal distribution, the median value  $\sigma^m = \exp(-\theta/2)$  is lower than one, and thus the median voter chooses a tax rate lower than  $\tau^*$ . The percentage deviation from the growth-maximizing tax rate increases with  $\theta$ , implying that greater inequality in relative factor endowments forces fiscal policy further away from the level that maximizes long-run growth.

This result quantifies how rising inequality—captured by an increase in the dispersion parameter  $\theta$ —amplifies the political economy distortion. The lognormal specification reflects the empirical regularity that labor income distributions are less skewed than wealth distributions. As  $\theta$  grows, the median voter's relative labor dependence  $\sigma^m$  rises exponentially, widening the gap between  $\tau^m$  and  $\tau^*$ . The quadratic growth losses from tax deviations (Proposition 5.1) then compound with  $\theta$ , creating a convex relationship between inequality and growth suppression. For sufficiently high  $\theta$ , the economy crosses the viability threshold in Proposition 5.2, entering an inequality-induced stagnation trap.

#### 6. Conclusions

This paper develops a dual public goods framework that extends the classic Alesina–Rodrik model by explicitly distinguishing between capital-augmenting (infrastructure) and labor-enhancing (human capital) public investments, both financed via a wealth tax. Our analysis shows that an optimal allocation of public spending exists—one that maximizes the net return on capital and thus supports higher long-run growth. In our model, when the government allocates tax revenues in accordance with the underlying technological parameters, the economy achieves a balanced growth rate that cannot be improved upon by any alternative fiscal composition.

Moreover, our political economy analysis reveals that distributional pressures may push the actual fiscal policy away from this growth-maximizing benchmark. In particular, if the median voter is relatively more dependent on labor income, the politically chosen tax rate tends to be higher than the optimal level, leading to distortions that reduce capital accumulation and slow economic growth. In extreme cases, such misaligned fiscal policies can even result in growth traps, where the net return on capital falls below the threshold required for sustainable accumulation.

Despite these insights, our results come with some limitations. The model abstracts from factors such as depreciation, government debt, and external shocks, and it relies on specific functional forms and equilibrium assumptions that may not capture all real-world complexities. Future research could extend the analysis by incorporating these additional dimensions and by examining the dynamic interplay between political institutions, inequality, and fiscal policy under uncertainty.

Our findings support the hypothesis that distributionally driven fiscal policies—when not insulated from political pressures—can impede long-run growth. Achieving optimal fiscal policy requires not only setting the right tax rate but also carefully allocating public spending in a way that reflects underlying production technologies rather than short-run redistributive concerns.



# 7. Appendix

## Appendix A: Proofs

# Proof of Proposition 3.3

To establish the result, we begin with the equilibrium gross rental rate,

$$r(\tau,\mu) = \alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{1-\alpha} L^{1-\alpha-\beta}.$$

Since  $\tau$ , L, A, and  $\alpha$  are exogenous (or determined elsewhere in the model), the dependence on the public spending composition is entirely captured by the term

$$\Phi(\mu) = \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta}.$$

It is convenient to take logarithms to facilitate differentiation. The log of  $\Phi(\mu)$  is

$$\ln \Phi(\mu) = \beta \ln \mu + (1 - \alpha - \beta) \ln(1 - \mu).$$

Differentiating with respect to  $\mu$  yields

$$\frac{d}{d\mu}\ln\Phi(\mu) = \frac{\beta}{\mu} - \frac{1-\alpha-\beta}{1-\mu}.$$

Setting this derivative equal to zero to find the critical point gives

$$\frac{\beta}{\mu^*} = \frac{1 - \alpha - \beta}{1 - \mu^*}.$$

A short algebraic manipulation leads to

$$\beta (1 - \mu^*) = (1 - \alpha - \beta) \mu^*,$$

so that

$$\mu^* = \frac{\beta}{1 - \alpha}.$$

The second derivative of  $\ln \Phi(\mu)$  is

$$\frac{d^2}{d\mu^2} \ln \Phi(\mu) = -\frac{\beta}{\mu^2} - \frac{1 - \alpha - \beta}{(1 - \mu)^2},$$

which is negative for all  $\mu \in (0,1)$ ; hence the critical point is a maximum.

### Proof of Proposition 3.6

[Proof of Proposition 3.6]

Since  $0 < 1 - \alpha < 1$ , the map  $\tau \mapsto \tau^{1-\alpha}$  is strictly concave. Hence  $\tau \mapsto F(\tau; \mu)$  is strictly concave, continuous, and satisfies  $\lim_{\tau \to 0^+} F(\tau; \mu) = 0$ ,  $\lim_{\tau \to \infty} F(\tau; \mu) = -\infty$ .

Setting  $\partial F/\partial \tau = 0$  gives

$$(1-\alpha)\alpha A\mu^{\beta}(1-\mu)^{1-\alpha-\beta}\tau^{-\alpha}L^{1-\alpha-\beta}=1,$$



hence the unique maximizer  $\tau^*(\mu) = \left[ (1-\alpha)\alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} L^{1-\alpha-\beta} \right]^{1/\alpha}$ .

By strict concavity, F rises from 0 at  $\tau = 0$  up to  $F(\tau^*; \mu)$  and then falls to  $-\infty$ . Under the assumption  $F(\tau^*; \mu) > 2\rho > \rho > 0$ , the horizontal lines  $y = \rho$  and  $y = 2\rho$  each intersect the graph of F exactly twice. The left-most intersection of  $y = \rho$  lies in  $(0, \tau^*)$ , we call it  $\tau_{\min}(\mu)$ . The right-most intersection of  $y = 2\rho$  lies in  $(\tau^*, \infty)$ , we call it  $\tau_{\text{máx}}(\mu)$ . No other intersections exist, so  $\rho < F(\tau; \mu) < 2\rho$  if and only if  $\tau_{\min}(\mu) < \tau < \tau_{\max}(\mu)$ .

Finally, since F depends on  $\mu$  only through the positive factor  $\Psi(\mu)$ , and  $\ln \Psi$  is strictly concave with a unique maximizer at  $\mu^* = \beta/(1-\alpha)$ , one deduces that for fixed  $\rho$  the solutions  $\tau_{\min}(\mu)$  and  $\tau_{\max}(\mu)$  are increasing for  $\mu < \mu^*$  and decreasing for  $\mu > \mu^*$ .

### Proof of Proposition 3.7

[Proof of Proposition 3.7] We begin with the balanced–growth rate

$$\gamma(\tau,\mu) = r(\tau,\mu) - \tau - \rho,$$

where

$$r(\tau,\mu) = \alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{1-\alpha} L^{1-\alpha-\beta}.$$

Since the parameters A, L,  $\alpha$ , and  $\beta$  are fixed, the dependence of  $\gamma$  on fiscal policy is entirely through  $\tau$  and  $\mu$ .

First, fix  $\mu$  and maximize  $\gamma(\tau,\mu)$  with respect to  $\tau$ . Differentiating with respect to  $\tau$  yields

$$\frac{\partial \gamma}{\partial \tau} = (1 - \alpha)\alpha A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} \tau^{-\alpha} L^{1 - \alpha - \beta} - 1.$$

Setting this derivative equal to zero gives the first-order condition

$$(1-\alpha)\alpha A\,\mu^\beta(1-\mu)^{\,1-\alpha-\beta}\,\tau^{-\alpha}L^{\,1-\alpha-\beta}=1.$$

Solving for  $\tau$  we obtain

$$\tau^*(\mu) = \left[\alpha A (1 - \alpha) L^{1 - \alpha - \beta} \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta}\right]^{1/\alpha}.$$

Next, we choose  $\mu$  to maximize the resulting growth rate. Notice that for any fixed  $\tau$ , the gross rental rate  $r(\tau, \mu)$  is proportional to the term

$$\Phi(\mu) = \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta}.$$

Taking logarithms yields

$$\ln \Phi(\mu) = \beta \ln \mu + (1 - \alpha - \beta) \ln(1 - \mu).$$

Differentiating with respect to  $\mu$  gives

$$\frac{d\ln\Phi(\mu)}{d\mu} = \frac{\beta}{\mu} - \frac{1-\alpha-\beta}{1-\mu}.$$

Setting this equal to zero we find

$$\frac{\beta}{\mu^*} = \frac{1 - \alpha - \beta}{1 - \mu^*} \implies \beta(1 - \mu^*) = (1 - \alpha - \beta)\mu^*,$$



which implies

$$\mu^* = \frac{\beta}{1 - \alpha}.$$

Substituting  $\mu^*$  into the expression for  $\tau^*(\mu)$  yields the unique growth–maximizing tax rate

$$\tau^* = [\alpha A (1 - \alpha) L^{1 - \alpha - \beta} (\mu^*)^{\beta} (1 - \mu^*)^{1 - \alpha - \beta}]^{1/\alpha}.$$

To show that  $(\tau^*, \mu^*)$  indeed maximizes  $\gamma$ , note that the second derivative of  $\gamma$  with respect to  $\tau$  is given by

$$\frac{\partial^2 \gamma}{\partial \tau^2} = -\alpha (1 - \alpha) \alpha A \,\mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} \,\tau^{-\alpha - 1} L^{1 - \alpha - \beta} < 0,$$

so that for any fixed  $\mu$ ,  $\gamma(\tau, \mu)$  is strictly concave in  $\tau$ . Similarly, the term  $\ln \Phi(\mu)$  is strictly concave in  $\mu$  because its second derivative is

$$\frac{d^2 \ln \Phi(\mu)}{d\mu^2} = -\frac{\beta}{\mu^2} - \frac{1 - \alpha - \beta}{(1 - \mu)^2} < 0.$$

Thus, the solution  $(\tau^*, \mu^*)$  is unique.

Finally, by the strict concavity of  $\gamma$  in  $\tau$  for fixed  $\mu$ , any deviation of the wealth tax rate from  $\tau^*$  reduces  $\gamma(\tau,\mu)$  by an amount that, to a first-order approximation, is quadratic in the deviation  $(\tau - \tau^*)$ . In other words, for  $\tau$  near  $\tau^*$ ,

$$\gamma(\tau, \mu^*) \approx \gamma(\tau^*, \mu^*) - \frac{1}{2} \left| \frac{\partial^2 \gamma}{\partial \tau^2} \right|_{\tau = \tau^*} \left| (\tau - \tau^*)^2, \right|$$

so that the loss in growth is convex in the deviation from the optimal tax rate.

This completes the proof.

### Proof of Proposition 5.1

Under the model's assumptions, each individual maximizes  $\widetilde{U}_i(\tau,\mu)$  with respect to  $\tau$ , and it has been derived that the optimal individual tax rate  $\tau_i$  is increasing in the relative factor endowment  $\sigma_i$ , so that

$$\frac{\partial \tau_i}{\partial \sigma_i} > 0.$$

In a political equilibrium where the median voter determines fiscal policy, the equilibrium tax rate  $\tau^m$  equals the tax rate preferred by the median voter, whose relative endowment is  $\sigma^m$ . For a representative agent with  $\sigma_i = 1$ , the growth-maximizing tax rate is  $\tau^*$ ; hence, if  $\sigma^m > 1$ , it must be that

$$\tau^m > \tau^*$$
.

Since the balanced-growth rate is given by

$$\gamma(\tau, \mu^*) = r(\tau, \mu^*) - \tau - \rho,$$

and because  $\gamma(\tau, \mu^*)$  is strictly concave in  $\tau$  with its maximum attained at  $\tau = \tau^*$ , any tax rate above  $\tau^*$  yields a lower growth rate. Thus, when the median voter's preference pushes the equilibrium tax rate to  $\tau^m > \tau^*$ , the economy's balanced-growth rate becomes

$$\gamma(\tau^m, \mu^*) < \gamma(\tau^*, \mu^*).$$



Moreover, since  $\tau^m$  is increasing in  $\sigma^m$ , an increase in the median relative factor endowment (i.e. a higher  $\sigma^m$ ) further increases  $\tau^m$ , thereby reducing  $\gamma(\tau^m, \mu^*)$ . This completes the demonstration that a higher median relative factor endowment (interpreted as greater inequality in the distribution of wealth versus labor income) induces a politically chosen tax rate above the growth-maximizing level and, as a result, lowers the long-run balanced-growth rate.

### Proof of Proposition 5.2

Since the individual's preferred tax rate  $\tau(\sigma)$  is strictly increasing in  $\sigma$ , one has  $\tau(1) = \tau^*$  and for any  $\sigma > 1$ ,  $\tau(\sigma) > \tau^*$ . The balanced-growth rate is given by

$$\gamma(\tau,\mu) = r(\tau,\mu) - \tau - \rho,$$

and, by the properties of the production function,  $\gamma(\tau,\mu)$  is maximized at  $\tau = \tau^*$  and is decreasing for  $\tau > \tau^*$ . Define  $\tau^z$  as the unique tax rate for which

$$r(\tau^z, \mu) - \tau^z = \rho.$$

Because  $\gamma(\tau,\mu)$  decreases for  $\tau > \tau^*$ , it follows that for any  $\tau > \tau^z$  the net return on capital is less than  $\rho$ , implying  $\gamma(\tau,\mu) < 0$ . Now, define  $\bar{\sigma}$  by the equation

$$\tau(\bar{\sigma}) = \tau^z$$
.

Then, if the median relative factor endowment satisfies  $\sigma^m > \bar{\sigma}$ , the equilibrium tax rate chosen by the median voter is

$$\tau^m = \tau(\sigma^m) > \tau(\bar{\sigma}) = \tau^z,$$

so that

$$r(\tau^m, \mu) - \tau^m < r(\tau^z, \mu) - \tau^z = \rho.$$

Thus, when  $\sigma^m > \bar{\sigma}$  the net return on capital falls below the discount rate  $\rho$ , implying that the balanced-growth rate  $\gamma(\tau^m, \mu)$  is negative; conversely, if  $\sigma^m < \bar{\sigma}$  then  $\tau^m < \tau^z$  and  $r(\tau^m, \mu) - \tau^m > 0$  $\rho$ , ensuring positive growth.

### Proof of Proposition 7

Since the individual preferred tax rate is a continuously differentiable function of  $\sigma$ , a firstorder Taylor expansion around  $\sigma = 1$  yields

$$\tau(\sigma) \approx \tau(1) + \tau'(1)(\sigma - 1).$$

By definition, the growth-maximizing tax rate is  $\tau^* = \tau(1)$ . Under the lognormal assumption with  $\ln \sigma \sim N(-\theta/2, \theta)$ , the median of  $\sigma$  is

$$\sigma^m = \exp\left(-\frac{\theta}{2}\right).$$

Thus, the median voter's preferred tax rate is approximately

$$\tau^m = \tau(\sigma^m) \approx \tau(1) + \tau'(1) \left( \exp\left(-\frac{\theta}{2}\right) - 1 \right).$$



Since  $\exp(-\theta/2) - 1 < 0$  for  $\theta > 0$ , it follows that  $\tau^m < \tau^*$ . The percentage deviation of the median tax rate from the growth-maximizing rate is then

$$\frac{\tau^* - \tau^m}{\tau^*} \approx -\frac{\tau'(1)}{\tau(1)} \left( \exp\left(-\frac{\theta}{2}\right) - 1 \right) = \frac{\tau'(1)}{\tau^*} \left[ 1 - \exp\left(-\frac{\theta}{2}\right) \right].$$

Because  $\tau'(1) > 0$ , the expression on the right is strictly increasing in  $\theta$  and vanishes when  $\theta = 0$ . This completes the proof.

## Appendix A.1: Detailed Derivation of Equilibrium Interest Rate

Starting from the firm's first-order condition for capital:

$$r_t = \alpha A K_{j,t}^{\alpha-1} (\mu \tau K_t)^{\beta} \left[ (1-\mu)\tau K_t L_{j,t} \right]^{1-\alpha-\beta}.$$

In symmetric equilibrium,  $K_{j,t} = K_t/m$  and  $L_{j,t} = L/m$ , where m is the number of firms. Substituting:

$$r_t = \alpha A \left(\frac{K_t}{m}\right)^{\alpha - 1} (\mu \tau K_t)^{\beta} \left[ (1 - \mu) \tau K_t \cdot \frac{L}{m} \right]^{1 - \alpha - \beta}.$$

Simplify exponents:

$$=\alpha A\mu^{\beta}(1-\mu)^{1-\alpha-\beta}\tau^{\beta+1-\alpha-\beta}K_t^{\alpha-1+\beta+1-\alpha-\beta}L^{1-\alpha-\beta}m^{-(\alpha-1+1-\alpha-\beta)}.$$

Cancel terms:

$$= \alpha A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} \tau^{1 - \alpha} L^{1 - \alpha - \beta}.$$

Thus,  $r_t$  is independent of m and scales with  $\tau^{1-\alpha}$ .

Interest Rate Responses: The equilibrium interest rate is given by:

$$r(\tau,\mu) = \alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{1-\alpha} L^{1-\alpha-\beta}$$
.

Derivative with Respect to Tax Rate  $(\tau)$ 

$$\frac{\partial r}{\partial \tau} = \alpha A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} (1 - \alpha) \tau^{-\alpha} L^{1 - \alpha - \beta} > 0,$$

since  $\alpha \in (0,1)$ , A > 0,  $\mu \in [0,1]$ , and L > 0. The positive response reflects that higher taxation expands public capital services  $(G_1)$  and labor-enhancing infrastructure  $(G_2)$ , raising capital's marginal productivity. Derivative with Respect to Allocation  $(\mu)$ 

$$\frac{\partial r}{\partial \mu} = \alpha A \tau^{1-\alpha} L^{1-\alpha-\beta} \left[ \beta \mu^{\beta-1} (1-\mu)^{1-\alpha-\beta} - (1-\alpha-\beta) \mu^{\beta} (1-\mu)^{-\alpha-\beta} \right].$$

Factorizing  $\mu^{\beta-1}(1-\mu)^{-\alpha-\beta}$ :

$$=\alpha A\tau^{1-\alpha}L^{1-\alpha-\beta}\mu^{\beta-1}(1-\mu)^{-\alpha-\beta}\left[\beta(1-\mu)-(1-\alpha-\beta)\mu\right].$$

Setting numerator to zero for critical point:

$$\beta(1-\mu) = (1-\alpha-\beta)\mu \quad \Rightarrow \quad \mu^* = \frac{\beta}{1-\alpha}.$$



Thus:

$$\frac{\partial r}{\partial \mu} \geq 0$$
 if  $\mu \geq \frac{\beta}{1-\alpha}$ .

When  $\mu < \beta/(1-\alpha)$ , infrastructure investment  $(G_1)$  is suboptimal - reallocating taxes toward  $G_1$ boosts capital productivity. Beyond  $\mu^*$ , excessive  $G_1$  crowds out  $G_2$ , reducing capital's efficiency gains.

Wage Rate Responses: The equilibrium wage rate is:

$$\omega(\tau,\mu) = (1 - \alpha - \beta)A\mu^{\beta}(1 - \mu)^{1 - \alpha - \beta}\tau^{1 - \alpha}L^{-(\alpha + \beta)}.$$

Derivative with Respect to Tax Rate  $(\tau)$ 

$$\frac{\partial \omega}{\partial \tau} = (1 - \alpha - \beta) A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} (1 - \alpha) \tau^{-\alpha} L^{-(\alpha + \beta)} > 0,$$

as all terms are positive. Higher  $\tau$  increases both public goods, enhancing labor productivity through  $G_2$ 's human capital effects. Derivative with Respect to Allocation  $(\mu)$ 

$$\frac{\partial \omega}{\partial \mu} = (1 - \alpha - \beta)A\tau^{1-\alpha}L^{-(\alpha+\beta)} \left[\beta\mu^{\beta-1}(1-\mu)^{1-\alpha-\beta} - (1-\alpha-\beta)\mu^{\beta}(1-\mu)^{-\alpha-\beta}\right].$$

Factorizing  $\mu^{\beta-1}(1-\mu)^{-\alpha-\beta}$ :

$$= (1 - \alpha - \beta)A\tau^{1-\alpha}L^{-(\alpha+\beta)}\mu^{\beta-1}(1-\mu)^{-\alpha-\beta}[\beta(1-\mu) - (1-\alpha-\beta)\mu].$$

Critical point:

$$\beta(1-\mu) = (1-\alpha-\beta)\mu \quad \Rightarrow \quad \mu^* = \frac{\beta}{1-\alpha}.$$

Thus:

$$\frac{\partial \omega}{\partial \mu} \gtrless 0 \quad \text{if} \quad \mu \lessgtr \frac{\beta}{1-\alpha}.$$

When  $\mu < \beta/1 - \alpha$ , labor-enhancing  $G_2$  is undersupplied - shifting funds to  $G_2$  raises wages. Beyond  $\mu^*$ , excessive  $G_1$  starves human capital investments, depressing labor productivity.

### Appendix A.2: Derivation of Fiscal Policy Multipliers

To quantify how factor prices respond to fiscal policy, we derive the elasticities of the interest rate (r) and wage rate (w) with respect to the tax rate  $(\tau)$  and public spending allocation  $(\mu)$ . These elasticities measure the percentage change in factor prices per 1% change in fiscal parameters, providing insights into policy effectiveness and distributional consequences.

Interest Rate Elasticities: Starting from the equilibrium interest rate:

$$r(\tau,\mu) = \alpha A \mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{1-\alpha} L^{1-\alpha-\beta},$$

take the natural logarithm to linearize the multiplicative relationships:

$$\ln r = \ln(\alpha A) + \beta \ln \mu + (1 - \alpha - \beta) \ln(1 - \mu) + (1 - \alpha) \ln \tau + (1 - \alpha - \beta) \ln L.$$



The tax rate elasticity  $\mathcal{E}_{r,\tau}$  is obtained by differentiating  $\ln r$  with respect to  $\ln \tau$ :

$$\mathcal{E}_{r,\tau} = \frac{\partial \ln r}{\partial \ln \tau} = 1 - \alpha.$$

This result reflects the exponential dependence of r on  $\tau^{1-\alpha}$ . A 1% increase in  $\tau$  expands public goods provision by  $(1-\alpha)$ %, proportionally enhancing capital productivity through both infrastructure  $(G_1)$  and human capital  $(G_2)$ .

For the allocation elasticity  $\mathcal{E}_{r,\mu}$ , differentiate  $\ln r$  with respect to  $\ln \mu$ :

$$\mathcal{E}_{r,\mu} = \frac{\partial \ln r}{\partial \ln \mu} = \beta - \frac{\mu(1 - \alpha - \beta)}{1 - \mu}.$$

The first term  $\beta$  captures infrastructure's direct contribution to capital productivity, while the second term  $-\frac{\mu(1-\alpha-\beta)}{1-\mu}$  represents the crowding-out effect on human capital. When  $\mu < \beta/(1-\alpha)$ , the positive infrastructure effect dominates, making  $\mathcal{E}_{r,\mu} > 0$ . Beyond this threshold, excessive infrastructure spending erodes labor-enhancing public goods, reducing capital's marginal product.

Wage Rate Elasticities: The equilibrium wage rate in log form is:

$$\ln \omega = \ln[(1 - \alpha - \beta)A] + \beta \ln \mu + (1 - \alpha - \beta) \ln(1 - \mu) + (1 - \alpha) \ln \tau - (\alpha + \beta) \ln L.$$

Differentiating with respect to  $\ln \tau$  gives:

$$\mathcal{E}_{\omega,\tau} = \frac{\partial \ln \omega}{\partial \ln \tau} = 1 - \alpha,$$

matching the tax elasticity of r. This symmetry arises because taxation scales both public goods equally, benefiting labor through human capital  $(G_2)$  and capital through infrastructure  $(G_1)$ .

The allocation elasticity for wages is:

$$\mathcal{E}_{\omega,\mu} = \frac{\partial \ln \omega}{\partial \ln \mu} = \beta - \frac{\mu(1 - \alpha - \beta)}{1 - \mu},$$

structurally identical to  $\mathcal{E}_{r,\mu}$  but with different economic implications.

### Appendix A.3: Sign of the Partial Derivatives

Step 1. Differentiating with respect to  $\sigma_i$ . Since only the first term in  $F(\tau, \sigma_i)$  depends on  $\sigma_i$ , we compute

$$\frac{\partial F}{\partial \sigma_i} = \frac{1}{\rho} \frac{\partial}{\partial \sigma_i} \left[ \frac{\omega_{\tau}(\tau, \mu) L \sigma_i}{\omega(\tau, \mu) L \sigma_i + \rho} \right].$$

For brevity, we write  $\omega_{\tau} \equiv \omega_{\tau}(\tau, \mu)$  and  $\omega \equiv \omega(\tau, \mu)$  (with the understanding that the dependence on  $\tau$  and  $\mu$  is implicit). Applying the quotient rule, we have

$$\frac{\partial}{\partial \sigma_i} \left[ \frac{\omega_\tau L \, \sigma_i}{\omega L \, \sigma_i + \rho} \right] = \frac{(\omega_\tau L)(\omega L \, \sigma_i + \rho) - (\omega_\tau L \, \sigma_i)(\omega L)}{(\omega L \, \sigma_i + \rho)^2} \,.$$

Notice that the numerator simplifies as

$$(\omega_{\tau}L)(\omega L \sigma_i + \rho) - (\omega_{\tau}L \sigma_i)(\omega L) = \omega_{\tau}L \rho.$$



Thus, we obtain

$$\frac{\partial}{\partial \sigma_i} \left[ \frac{\omega_\tau L \, \sigma_i}{\omega L \, \sigma_i + \rho} \right] = \frac{\omega_\tau L \, \rho}{\left(\omega L \, \sigma_i + \rho\right)^2} \, .$$

Hence,

$$\frac{\partial F}{\partial \sigma_i} = \frac{1}{\rho} \cdot \frac{\omega_\tau L \rho}{\left(\omega L \sigma_i + \rho\right)^2} = \frac{\omega_\tau L}{\left(\omega L \sigma_i + \rho\right)^2}.$$

Since  $\omega_{\tau} > 0$  and L > 0, it follows that

$$\frac{\partial F}{\partial \sigma_i} > 0$$
.

# Step 2. Detailed Derivation of the Sign of $\partial F/\partial \tau$ . Recall that

$$F(\tau, \sigma_i) = \frac{1}{\rho} \frac{\omega_{\tau}(\tau, \mu) L \sigma_i}{\omega(\tau, \mu) L \sigma_i + \rho} + \frac{1}{\rho^2} \Big[ r_{\tau}(\tau, \mu) - 1 \Big],$$

where by definition

$$\omega(\tau,\mu) = C_1 \tau^{1-\alpha}$$
 and  $r(\tau,\mu) = C_2 \tau^{1-\alpha}$ ,

with

$$C_1 = (1 - \alpha - \beta) A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} L^{-(\alpha + \beta)}$$
 and  $C_2 = \alpha A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} L^{1 - \alpha - \beta}$ ,

which are positive constants with respect to  $\tau$ . Hence, differentiating with respect to  $\tau$  yields

$$\omega_{\tau}(\tau,\mu) = \frac{\partial \omega(\tau,\mu)}{\partial \tau} = (1-\alpha)C_1 \tau^{-\alpha} = (1-\alpha)\frac{\omega(\tau,\mu)}{\tau},$$

and similarly,

$$r_{\tau}(\tau,\mu) = (1-\alpha)\frac{r(\tau,\mu)}{\tau}$$
.

Next, we compute the second derivative of  $r(\tau, \mu)$  with respect to  $\tau$ . Differentiating

$$r_{\tau}(\tau,\mu) = (1-\alpha)\frac{r(\tau,\mu)}{\tau},$$

we have by the quotient rule:

$$r_{\tau\tau}(\tau,\mu) = \frac{\partial}{\partial \tau} \left[ (1-\alpha) \frac{r(\tau,\mu)}{\tau} \right] = (1-\alpha) \frac{r_{\tau}(\tau,\mu) \tau - r(\tau,\mu)}{\tau^2}.$$

Substitute the expression for  $r_{\tau}(\tau, \mu)$ :

$$r_{\tau}(\tau,\mu) \tau - r(\tau,\mu) = (1-\alpha) \frac{r(\tau,\mu)}{\tau} \tau - r(\tau,\mu)$$
$$= (1-\alpha)r(\tau,\mu) - r(\tau,\mu)$$
$$= [(1-\alpha)-1]r(\tau,\mu)$$
$$= -\alpha r(\tau,\mu).$$

Thus,

$$r_{\tau\tau}(\tau,\mu) = -\alpha(1-\alpha)\frac{r(\tau,\mu)}{\tau^2}$$
.



Now, define

$$Q(\tau) = \frac{\omega_{\tau}(\tau, \mu) L \sigma_{i}}{\omega(\tau, \mu) L \sigma_{i} + \rho}.$$

Then, the first term of  $F(\tau, \sigma_i)$  is  $F_1(\tau, \sigma_i) = \frac{1}{\rho} Q(\tau)$  and the second term is  $F_2(\tau) = \frac{1}{\rho^2} \Big[ r_{\tau}(\tau, \mu) - 1 \Big]$ . Differentiating F with respect to  $\tau$ , we obtain

$$\frac{\partial F}{\partial \tau} = \frac{1}{\rho} Q'(\tau) + \frac{1}{\rho^2} r_{\tau\tau}(\tau, \mu).$$

We now compute  $Q'(\tau)$ . Write

$$Q(\tau) = \frac{A(\tau)}{B(\tau)},$$

with

$$A(\tau) = \omega_{\tau}(\tau, \mu) L \sigma_i$$
 and  $B(\tau) = \omega(\tau, \mu) L \sigma_i + \rho$ .

Then, by the quotient rule,

$$Q'(\tau) = \frac{A'(\tau)B(\tau) - A(\tau)B'(\tau)}{[B(\tau)]^2}.$$

First, differentiate  $A(\tau)$ :

$$A(\tau) = \omega_{\tau}(\tau, \mu) L \sigma_i = (1 - \alpha) \frac{\omega(\tau, \mu)}{\tau} L \sigma_i,$$

so that using the quotient rule on  $\frac{\omega(\tau,\mu)}{\tau}$ ,

$$\frac{d}{d\tau} \left[ \frac{\omega(\tau,\mu)}{\tau} \right] = \frac{\omega_{\tau}(\tau,\mu) \tau - \omega(\tau,\mu)}{\tau^2} .$$

Thus,

$$A'(\tau) = (1 - \alpha)L \,\sigma_i \, \frac{\omega_{\tau}(\tau, \mu) \,\tau - \omega(\tau, \mu)}{\tau^2} \,.$$

Substitute  $\omega_{\tau}(\tau,\mu) = (1-\alpha)\frac{\omega(\tau,\mu)}{\tau}$ :

$$\omega_{\tau}(\tau,\mu)\tau - \omega(\tau,\mu) = (1-\alpha)\omega(\tau,\mu) - \omega(\tau,\mu) = -\alpha\omega(\tau,\mu).$$

so that

$$A'(\tau) = -\alpha(1-\alpha)L\,\sigma_i\,\frac{\omega(\tau,\mu)}{\tau^2}\,.$$

Next, differentiate

$$B(\tau) = \omega(\tau, \mu) L \, \sigma_i + \rho,$$

which gives

$$B'(\tau) = \omega_{\tau}(\tau, \mu) L \, \sigma_i = A(\tau) \, .$$

Therefore,

$$Q'(\tau) = \frac{-\alpha(1-\alpha)L\,\sigma_i\,\frac{\omega(\tau,\mu)}{\tau^2}\,B(\tau) - [A(\tau)]^2}{[B(\tau)]^2}\,.$$



Since all the parameters  $(\omega(\tau,\mu), L, \sigma_i, \text{ and } \tau)$  and  $B(\tau)$  are positive, it follows that

$$Q'(\tau) < 0$$
.

Also, note that the second term in  $\partial F/\partial \tau$  is

$$\frac{1}{\rho^2} r_{\tau\tau}(\tau, \mu) = -\frac{\alpha(1-\alpha)}{\rho^2} \frac{r(\tau, \mu)}{\tau^2} ,$$

which is clearly negative.

Hence, both terms in  $\frac{\partial F}{\partial \tau}$  are negative, so that

$$\frac{\partial F}{\partial \tau} = \frac{1}{\rho} Q'(\tau) + \frac{1}{\rho^2} r_{\tau\tau}(\tau, \mu) < 0.$$

Finally, by the implicit function theorem,

$$\frac{d\tau}{d\sigma_i} = -\frac{\partial F/\partial \sigma_i}{\partial F/\partial \tau} .$$

In the extended Alesina–Rodrik framework, define the balanced-growth rate as

$$\gamma(\tau,\mu) = \alpha A \,\mu^{\beta} (1-\mu)^{1-\alpha-\beta} \tau^{1-\alpha} L^{1-\alpha-\beta} - \tau - \rho,$$

and the elasticity of the balanced-growth rate with respect to the wealth tax as

$$\mathcal{E}_{\gamma,\tau} = \frac{\partial \gamma(\tau,\mu)}{\partial \tau} \frac{\tau}{\gamma(\tau,\mu)}.$$

Then, for any given tax rate  $\tau$  and exogenous parameters,  $\mathcal{E}_{\gamma,\tau}$  is maximized with respect to the public spending composition parameter  $\mu$  at the unique value

$$\mu^* = \frac{\beta}{1 - \alpha}.$$

In other words, the marginal growth benefits of increasing the wealth tax are highest when public spending is allocated optimally in the ratio  $\mu^* = \beta/(1-\alpha)$ .

To begin, denote

$$R(\mu) = \alpha A \mu^{\beta} (1 - \mu)^{1 - \alpha - \beta} L^{1 - \alpha - \beta},$$

so that the balanced-growth rate can be written as

$$\gamma(\tau,\mu) = R(\mu) \, \tau^{1-\alpha} - \tau - \rho.$$

Differentiating  $\gamma(\tau,\mu)$  with respect to  $\tau$  while holding  $\mu$  constant yields

$$\frac{\partial \gamma}{\partial \tau} = (1 - \alpha) R(\mu) \tau^{-\alpha} - 1.$$

Thus, the elasticity of  $\gamma$  with respect to  $\tau$  is

$$\mathcal{E}_{\gamma,\tau} = \frac{\partial \gamma}{\partial \tau} \frac{\tau}{\gamma(\tau,\mu)} = \frac{\left[ (1-\alpha)R(\mu)\tau^{-\alpha} - 1 \right]\tau}{R(\mu)\tau^{1-\alpha} - \tau - \rho}.$$



For fixed  $\tau$  and  $\rho$ , the only dependence on  $\mu$  in  $\mathcal{E}_{\gamma,\tau}$  enters through  $R(\mu)$ . Since the expression

$$R(\mu) = \alpha A L^{1-\alpha-\beta} \, \mu^{\beta} (1-\mu)^{1-\alpha-\beta}$$

is strictly increasing in its multiplicative factor  $\mu^{\beta}(1-\mu)^{1-\alpha-\beta}$ , the elasticity  $\mathcal{E}_{\gamma,\tau}$  is maximized when  $R(\mu)$  is maximized. Taking logarithms we obtain

$$\ln R(\mu) = \ln \left[ \alpha A L^{1-\alpha-\beta} \right] + \beta \ln \mu + (1-\alpha-\beta) \ln (1-\mu).$$

Differentiating with respect to  $\mu$  gives

$$\frac{d\ln R(\mu)}{d\mu} = \frac{\beta}{\mu} - \frac{1 - \alpha - \beta}{1 - \mu}.$$

Setting the derivative equal to zero, we have

$$\frac{\beta}{\mu} = \frac{1 - \alpha - \beta}{1 - \mu},$$

which, upon cross-multiplying and rearranging, yields

$$\beta(1-\mu) = (1-\alpha-\beta)\mu.$$

Solving for  $\mu$ , we obtain

$$\mu^* = \frac{\beta}{1 - \alpha}$$
.

Since the second derivative of  $\ln R(\mu)$  with respect to  $\mu$  is negative, this critical value is the unique maximizer of  $R(\mu)$ , and hence the unique maximizer of  $\mathcal{E}_{\gamma,\tau}$ . Therefore, for any fixed  $\tau$  and  $\rho$ , the elasticity of the balanced-growth rate with respect to the wealth tax is maximized at  $\mu^* = \beta/(1-\alpha)$ .

The sensitivity of the economy's growth rate to a change in the wealth tax is greatest when public spending is allocated in an optimal proportion between infrastructure and human-capital investments. When the government chooses  $\mu = \mu^*$ , the positive productivity effect of additional public spending fully offsets the distortionary impact of taxation, thereby maximizing the marginal benefit of raising  $\tau$ . Deviations from this optimal allocation diminish the multiplier effect of the tax rate on growth, so that even a marginal increase in  $\tau$  yields a smaller boost in the net return on capital and hence a lower growth rate. This finding provides a novel insight into the importance of fiscal composition: not only does the tax rate matter for growth, but the allocation of public spending is crucial in determining the efficacy of taxation as an instrument for promoting long-run economic growth.



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