



## Empirical determinants of financial fragility. The case of Colombian firms

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### Abstract

Hyman Minsky's financial instability hypothesis provides a theoretical framework to understand the emergence of endogenous crises in modern economies and how capital flows amplify accumulated imbalances and exacerbate financial constraints in economic units. This inquiry operationalizes the Financial Instability Hypothesis within the Colombian non-listed manufacturing sector through the estimation of discrete-state dynamics and distributional sensitivities. The methodological design constructs two distinct fragility taxonomies to interrogate the determinants of the Hedge, Speculative, and Ponzi classifications. The first specification applies an open-economy cash-flow model derived from Castro (2011), which explicitly internalizes the valuation effects of nominal exchange rate fluctuations on debt service obligations. The second taxonomy, grounded in Nishi (2019), evaluates solvency through the interaction of a flow-based profitability margin and a stock-based liquid asset buffer. To parse the transmission of meso-level economic impulses, the analysis deploys multinomial logit models equipped with a Mundlak correction for correlated random effects alongside recentered influence function regressions. Estimation outputs from the first model confirm that the deterioration of the interest coverage ratio functions as the primary determinant to the Ponzi state, while pre-existing dependence on imported capital acts as a specific transmission channel for currency shocks. The margin-of-safety specification reveals that stock-based liquidity buffers absorb solvency shocks effectively, rendering specific currency exposure variables redundant as predictors of distress. Finally, the dynamic analysis uncovers a temporal asymmetry where contemporaneous sectoral expansions ameliorate immediate default risk through the revenue channel, whereas lagged growth accumulation is associated with the endogenous generation of future fragility. This validates the core thesis of Minsky's framework: that stability breeds instability.

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## 1. Introduction

The recurrent character of financial upheavals and the cyclical incidence of systemic financial dislocations, particularly within emerging market economies, continues to present a formidable analytical problem for orthodox economic theory. While the causal locus of recent instability episodes frequently resides within the non-financial corporate sector, the precise propagation mechanisms remain a subject of intense analytical debate. The Colombian economic structure, with its deep articulation with international capital markets and capital circuits and its structural dependence on imported capital equipment, offers a pertinent yet seldom scrutinized case for examination. Conventional asset-pricing and credit-risk architectures, typically specified for publicly-traded corporations in informationally efficient markets, are methodologically inadequate for this context. The informational opacity of the capital accounts of non-listed manufacturing entities, coupled with the idiosyncratic financial constraints they confront, compels a more granular, theoretically-grounded investigation into the endogenous generation of financial fragility.

Although an extensive corpus of scholarship examines the macroeconomic and microeconomic antecedents of financial instability, a methodological void persists. Prior inquiries have operationalized the principles of Hyman Minsky across diverse economic settings, from developing nations to advanced economies. What remains absent is a systematic empirical application of these principles to the specific institutional and operational context of Colombian non-listed manufacturing corporations.

In Minsky's theoretical framework each balance sheet constitutes a formal registry of a network pecuniary obligations, wherein capital assets are primary instruments for the pursuing of private wealth and the generation of monetary returns. In other words, it forms a sophisticated financial superstructure that enables indirect ownership of wealth.

This system requires the execution of monetary transfers, both ingress and egress (in/out). In particular, commercial paper of short duration, bonds, deposit certificates, insurance policies, and shares of joint-stock enterprises - operate as mechanisms through which control over capital assets shall be secured. Such instruments must invariably impose upon their holders specific obligations to remit cash at fixed temporal intervals or upon the occurrence of financial contingencies.

The employment of said instruments necessitate the procurement of cash for the fulfillment of these obligations. Such monetary resources must be obtained through diverse channels, viz.: existing pecuniary reserves, income derived from wages and commercial profits, proceeds accruing from extant financial contracts, the liquidation of tangible assets and monetary instruments, the issuance of new debt obligations, the creation of additional monetary units.

The concatenation of financial obligations establishes a temporal nexus between economic activities past, present, and future. Prior pecuniary commitments generate both immediate and prospective monetary outflows, while extant liabilities give rise to future obligations of payment.

According to [Minsky \(1991\)](#), the accounting structure of a commercial enterprise should be interpreted through examination of these monetary transfers. The balance sheet of such an enterprise presents, on one side, a precise enumeration of physical and financial assets, whilst the other side shall catalog all liabilities. The statement of income furnishes an exact account of

uses and sources of cash. Said income statement constitutes a complete registry of net monetary influx derived from operations, to wit: the differential between revenues from sales and immediate expenditures is the most comprehensive measure of commercial profits. These gross profits are equivalent to the monetary flows generated through the firm's position in the market, its capital assets, and its organizational constitution.

In Minskyan nomenclature, "gross profit" is synonymous with "gross capital income," derived from "total operational receipts minus current labor and material costs." This gross capital income is allocated to debt servicing (principal and interest), discharge of taxation duties, and proprietor remuneration. The level of profits remains the principal determinant of systematic commercial behavior, as it dictates both the capacity for debt service and the propensity for further investment.

Moreover, Minsky succinctly describes the nature of non-operational cash inflows. Another category of cash flow to a unit is from financial instruments that it owns; these cash flows represent fulfillment of contractual obligations by other parties. Beyond the cash flows derived from gross profits and contractual fulfillments, an entity generates cash through the disposition of physical or financial assets or by collateralizing assets or future revenue streams, constituting the counterpart to assets on a firm's balance sheet. These represent obligations for future disbursements, dedicated to both repaying and servicing debt. Cash to meet these payment commitments can be sourced from various channels: gross profit cash flow, existing liquid reserves, asset liquidation, or additional borrowing.

In contrast to the Walrasian framework, wherein money assumes a neutral role, Minsky's theoretical framework ascribes a distinctive function to monetary instruments. The conventional Walrasian perspective assumes an economy in equilibrium, characterized by perfect substitutability of goods and the absence of transactions external to equilibrium. This model fails to account for empirical phenomena of considerable import, to wit: transactions costs and the fundamental uncertainty of future events ([Van den Hauwe, 2020](#)).

Minsky identifies two principal defects in the investment expansion. First stands its speculative character. In the general euphoria commercial enterprises assume debt commitments at a pace that outstrips their profits. These enterprises proceed from conventional methods of finance to speculative arrangements, wherein fresh debt services existing obligations, and thence to ultra-speculative or Ponzi schemes, where debt maintains interest payments alone. Indeed, an internal evolutionary process diminishes safety margins absent any requirement for excessive optimism. The estimation of future payment capacity during cyclical expansion may be deemed a rational response to historical experience. Sanguine expectations of the ability to meet cash commitments in a cyclical expansion can be interpreted as a reaction to the evaluation of past events, as expressed in higher probabilities of success. The persistence of the boom inexorably begets either constraints in the financial mechanism or inflationary pressures in commodity markets. These developments compel monetary authorities to implement contractionary policies to restore equilibrium ends the boom, and the investment-profit chain. The subsequent elevation of interest rates terminates the expansion, whereupon the investment-profit chain reverts to a downward spiral.

More generally, the inevitability of the upper turning point is explained in Minsky's theory, which delineates that a crisis manifests under three possible circumstances: when the burden of finance exceeds the capacity for payment, when liquidity preference strengthens, or when there is a shortfall in realized income flows relative to expectations.

A plethora of research has been conducted to elucidate the factors that explains the genesis of financial instability through both macroeconomic and microeconomic optics. Concurrent investigations have adapted these Minskian precepts to varied economic geographies, including both mature economies (Keen, 2020; Rammelt, 2019) and emerging markets (Médici, 2020). The analytical distinction between these economic types is paramount. Models calibrated for developed markets typically focus on endogenous cycle generation within financially deep and institutionally stable settings. In contrast, analyses of emerging markets must contend with institutional vulnerabilities that are often absent in their developed counterparts. A primary distinction pertains to the structural currency mismatch; corporations in developing nations frequently accumulate liabilities denominated in a foreign reserve currency while their revenue streams are denominated in the local currency. This condition, compounded by the volatility of international capital flows, exposes these firms to acute balance sheet effects from nominal exchange rate fluctuations and to severe refinancing risks during global "sudden stops".

Macroeconomic inquiries, such as those by Feijó et al. (2021) and Eazzazi (2022), typically examine aggregate balance sheet phenomena, quantifying the relationship between national credit cycles and capital formation dynamics. Such top-down analyses, while valuable for identifying systemic trends, necessarily abstract from the granular, firm-level decision-making processes where financial commitments are originally contracted. Concurrently, microeconomic explorations, exemplified by the work of Apreda (2012), pursue a bottom-up approach, focusing on the operationalization of Minskian classifications through the inspection of corporate cash-flow adequacy and liability structures. These firm-centric studies, however, often proceed within a closed-economy assumption and omit the transmission mechanisms of external financial shocks.

Notwithstanding, there is a dearth of research on the application of Minsky's principles in the Colombian economy that appears to be a relatively understudied area. The Colombian economic structure presents a distinct confluence of these vulnerabilities that general emerging-market models do not fully capture. The nation's substantial cohort of non-listed, capital-intensive manufacturing enterprises confronts a dual exposure: they procure essential capital equipment from international markets because of dependency on imported goods and frequently finance these acquisitions through foreign currency-denominated debt. This configuration creates a direct transmission channel through which nominal exchange rate fluctuations propagate directly into both the cost of capital formation and the real debt service burden, a dynamic that remains insufficiently studied in the extant literature.

To advance this empirical and theoretical line of inquiry, this study proceeds from the foundational architecture of Hyman Minsky, whose work supplies a uniquely suitable optic for the analysis of the inner mechanics of a capitalist economy structured by debt. Although an extensive corpus of scholarship examines the macroeconomic and microeconomic antecedents of financial instability, a methodological void persists. Prior inquiries have operationalized the principles

of Hyman Minsky across diverse economic settings, from developing nations to advanced economies. What remains absent is a systematic empirical application of these principles to the specific institutional and operational context of Colombian non-listed manufacturing corporations. The present inquiry advances the scholarship through three discrete analytical thrusts. First, it constructs a rigorous classification methodology to sort firms into Hedge, Speculative, and Ponzi postures by adapting advanced theoretical constructions to the data constraints common to private entities. A primary analytical contribution materializes in the construction of a robust, dual-classification taxonomy that sorts enterprises into Hedge, Speculative, and Ponzi financial postures. This procedure operationalizes three discrete theoretical architectures and recalibrates them for the informational limitations characteristic of private corporations. The principal classification method proceeds from the open-economy, cash-flow mechanics of [Fama \(1983\)](#), a system that internalizes nominal exchange rate dynamics within the calculus of debt service capacity. The second specification adopts the margins-of-safety paradigm of [Nishi \(2019\)](#), which appraises fragility by reference to a firm's flow-solvency margin and its stock-based liquidity cushion. This approach addresses the absence of market-based valuations for non-listed companies; it substitutes auditable book values and derives volatility inputs from high-frequency operational cash flow data.

A further contribution of this inquiry is supplied through the formulation of a linear probabilistic model engineered to isolate the specific microeconomic antecedents that condition a firm's position on the fragility continuum. A specification is utilized to compute the effect of determinants that encompass metrics of operational efficiency, investment conduct congruent with euphoric cycle phenomena, and explicit financial policy decisions.

## 2. Methodology

In the first stage of this research, a comprehensive dataset was compiled by collecting balance sheets and financial statements from 3376 Colombian manufacturing companies across a six-year period from 2014 to 2019.

[Table 1](#) describes financial metrics used to evaluate a company's performance and financial health. The descriptions and rationale are framed specifically for the sample of capital-intensive manufacturing firms that rely on imported capital goods. This table outlines the key independent variables selected to predict a firm's transition into fragile financial states within a multinomial logit panel model. Each indicator is designed to act as an empirical proxy for the framework described by Hyman Minsky, for open, emerging economies. This table contains a comprehensive exposition of the construction and theoretical underpinnings of the variables employed in the econometric models. The structure is bifurcated: the first section delineates the firm-level regressors for the fragility transition models (under the Castro and Nishi taxonomies); the second section defines the meso-level and firm-level variables that function as inputs in the analysis of business cycle effects on corporate fragility (the [Davis et al. \(2019\)](#) adaptation).

**Table 1**

Operational Definition and Theoretical Foundation of Econometric Variables

**A. Variables for the Fragility Transition Models (Castro & Nishi Frameworks)**

Variable Name		Formula & Operational Definition	Economic Rationale for Inclusion (Castro 2011 Framework)	Directional Hypothesis & Theoretical Foundation (Nishi 2019 Framework)
Asset (TANG)	Tangibility	$PPE_{net} / TA$ . A ratio where the numerator, $PPE_{net}$ , is the book value of Property, Plant, and Equipment net of accumulated depreciation, and the denominator, $TA$ , is the book value of Total Assets.	Tangible assets supply a stable valuation anchor during episodes of exchange rate volatility. Their value in local currency terms has less susceptibility to abrupt revaluations compared to financial or intangible assets, which offers a buffer on the asset side of the balance sheet that can counteract currency-induced expansions of foreign-denominated liabilities.	Asset tangibility is the principal form of collateral, a mechanism that minimizes informational asymmetries and agency costs in credit markets. A superior collateral base grants a firm preferential access to debt capital and lowers its cost of funds, a process that directly fortifies its balance sheet and constitutes a fundamental stock-based margin of safety.
Imported Capital Share (IMPCAP_pre)		$\frac{\sum_{t=t_0-n}^{t_0-1} CAPIM_{it}}{\sum_{t=t_0-n}^{t_0-1} CAPEX_{it}}$ The ratio of Capital Imports ( $CAPIM_{it}$ ) to Gross Capital Expenditures ( $CAPEX_{it}$ ) averaged over a pre-sample period ( $t_0 - n$ to $t_0 - 1$ ) to capture a pre-determined structural characteristic.	This variable is a direct operationalization of a pivotal transmission channel in the open-economy Minsky model. It specifies a firm's structural vulnerability to nominal exchange rate depreciations, which concurrently inflate the local-currency cost of new investment and the service burden of any foreign-denominated debt used for its finance.	While Nishi's model is currency-agnostic, a structural dependence on imported capital often correlates with large, lumpy, and technologically specific investment projects. Such projects place considerable strain on a firm's internal finance capacity and oblige recourse to external debt, a sequence that erodes both the flow-solvency margin ( $\mu$ ) and the liquid asset kicker ( $\eta$ ).
Firm Age (AGE)		$t_{current} - t_{inception}$ . The number of years elapsed since the firm's formal incorporation $t_{inception}$ up to the current year of observation $t_{current}$ .	Firm longevity can be a proxy for accumulated reputational capital in credit markets. More established firms may have superior access to currency risk-mitigation instruments and more stable relationships with international lenders, which can attenuate the open-economy financial risks articulated by Castro.	Age is a proxy for accumulated institutional knowledge and financial stability. Mature firms have navigated multiple credit cycles and have had more time for the accumulation of retained earnings and other balance-sheet buffers. They operate with a lower degree of informational opacity, which translates into superior and more stable margins of safety.

Variable Name	Formula & Operational Definition	Economic Rationale for Inclusion (Castro 2011 Framework)	Directional Hypothesis & Theoretical Foundation (Nishi 2019 Framework)
<b>Firm Size (SIZE)</b>	$\ln(N_{emp})$ . The natural logarithm of the total number of employees $N_{emp}$ , an action that functions as a proxy for the firm's operational scale.	Larger firms often command greater bargaining power with foreign suppliers and may operate with a naturally hedged portfolio of international assets and liabilities. Their scale may also grant them access to more sophisticated and less costly financial instruments for the management of currency exposure.	Corporate scale is associated with operational diversification and preferential access to capital markets. These advantages stabilize aggregate cash flows and lower the cost of external finance, conditions that directly contribute to more robust flow-solvency margins ( $\mu$ ) and a superior capacity to maintain precautionary liquidity buffers ( $\eta$ )
<b>Interest Coverage Ratio (ICR_avg)</b>	$\frac{1}{3} \sum_{k=0}^2 \frac{EBIT_{t-k}}{IE_{t-k}}$ . A three-year moving average of the ratio of Earnings Before Interest and Taxes (EBIT) to Interest Expense (IE), designed to capture the firm's structural debt service capacity.	In Castro's framework, which centers on cash flows, a deteriorating ICR is the primary symptom of a firm's inability to meet its debt service obligations, V. A persistently low ICR, especially in the presence of foreign currency debt, signals an imminent breach of the solvency conditions that define the Hedge and Speculative states.	The ICR is the empirical core of the Minskyan taxonomy and the direct measure of the flow-solvency margin ( $\mu$ ). An ICR consistently below unity is the defining characteristic of a Ponzi unit. A value comfortably above one is a necessary, though not sufficient, condition for a Hedge classification in Nishi's dual-margin system.
<b>Cash Holdings Ratio (CASH/TA)</b>	$\frac{C}{TA}$ . A ratio where the numerator, C, is the firm's stock of Cash and Cash Equivalents, and the denominator, TA, is Total Assets.	While Castro's flow-based model does not explicitly feature precautionary cash holdings, a large stock of domestic currency (cash) can function as a self-insurance mechanism against an unexpected and abrupt revaluation of foreign currency liabilities. This allows the firm to service its debt without recourse to asset fire sales.	This ratio is a direct empirical proxy for the "liquid asset kicker" ( $\eta$ ), one of the two pillars of Nishi's taxonomy. It represents a stock-based margin of safety, a liquidity buffer held explicitly for the absorption of unforeseen cash-flow shortfalls and for the preservation of financial independence from capricious credit markets.



Variable Name		Formula & Operational Definition	Economic Rationale for Inclusion (Castro 2011 Framework)	Directional Hypothesis & Theoretical Foundation (Nishi 2019 Framework)
<b>Retention (RET_avg)</b>	<b>Rate</b>	$1 - \frac{1}{3} \sum_{k=0}^2 \frac{DIV_{t-k}}{NI_{t-k}}$ One minus a three-year moving average of the dividend payout ratio (Dividends, DIV, over Net Income, NI). It captures the proportion of earnings reinvested in the firm.	In an open economy, retained earnings denominated in local currency are the safest source of funds for capital expenditures, particularly for imported capital goods. A superior retention rate lessens the firm's need for external debt, especially foreign-denominated debt, a condition that contracts its exposure to currency and rollover risks.	A superior retention rate is the engine of internal capital accumulation, which directly fortifies a firm's equity base and its liquidity position. This process of self-finance is central to the construction and maintenance of Nishi's margins of safety, as it strengthens the balance sheet without the introduction of new, rigid financial claims.
<b>Short-Term Debt Share (ST_SHARE)</b>	<b>Share</b>	$\frac{STD}{TD}$ The ratio of Short-Term Debt (STD, liabilities with maturity under one year) to Total Debt (TD).	In an open economy with volatile capital flows, a concentration of short-term debt exposes a firm to acute rollover risk. A "sudden stop" in capital inflows can make the refinance of maturing foreign-currency debt impossible or prohibitively expensive, a dynamic that can rapidly propel a solvent firm into a liquidity crisis.	A superior proportion of short-term debt signifies a precarious financial posture. The firm needs a constant need for market validation and an absence of long-term funding certainty. This structure is fundamentally at odds with the concept of a margin of safety, as it maximizes exposure to liquidity shocks and interest rate volatility.
<b>Long-Run (LEV_LR)</b>	<b>Leverage</b>	$\frac{1}{n} \sum_{t=t_0-n}^{t_0-1} \frac{TD_{it}}{TA_{it}}$ The pre-sample average of the firm's total debt (TD) to total assets (TA) ratio, computed to capture its structural financial policy or "type" regarding indebtedness.	A firm with a historically elevated level of leverage, particularly if a component is foreign-denominated, enters the sample period with a structurally thinner equity cushion. This pre-existing condition makes it more susceptible to the adverse balance-sheet effects of a currency depreciation, as the shock operates on a larger base of liabilities.	Minsky's hypothesis describes a cumulative process. A firm's long-run leverage policy reveals its structural risk appetite. A perpetually high-leverage firm operates with a compressed margin of safety by design. This predisposition makes it more likely to breach solvency thresholds when any adverse operational or financial shock appears.



## B. Variables for the Sectoral Business Cycle Model (Davis & de Souza Adaptation)

Variable Name	Formula & Operational Definition	Economic Rationale for Inclusion	Directional Hypothesis & Theoretical Foundation
<b>Sectoral Output Gap (Output_Gap_Sectoral)</b>	$g_{st} = y_{st} - \tau_{st}$ The cyclical component ( $g_{st}$ ) of the logarithm of a sector's real gross value added ( $y_{st}$ ), obtained after the extraction of its secular trend component ( $\tau_{st}$ ) via a Hodrick-Prescott filter.	This variable is the primary exogenous impulse in the model. It captures meso-level cyclical fluctuations that are external to the decisions of any individual firm but directly influences its operating environment. The foundational identification assumption is that a single firm's performance cannot alter the cyclical trajectory of its entire sector. A positive gap denotes a sectoral expansion; a negative gap signifies a contraction.	A positive sectoral output gap (an expansionary phase) is expected to translate into superior revenues and cash flows for firms within that sector. This facilitates the fulfillment of their financial obligations. The magnitude of the estimated coefficient is the central object of scrutiny; a parameter of small economic significance would imply that short-run cyclical phenomena are not the principal mechanism of the fragility dynamics.
<b>Sectoral Growth Rate (Sectoral_Growth)</b>	$\Delta \%GVA_{st} = \frac{GVA_{st} - GVA_{st-1}}{GVA_{st-1}}$ The annual percentage change in the real Gross Value Added (GVA) of sector $s$	This metric offers an alternative and more direct measure of a sector's economic momentum. In contrast to the statistically constructed output gap, the growth rate captures the immediate year-over-year variation in economic activity. Its inclusion is for the purpose of a consistency and robustness check on the findings obtained with the output gap variable, as both metrics should theoretically capture the same underlying pro-cyclical pressures on firm-level financial health.	The theoretical foundation is analogous to that for the output gap. A superior rate of sectoral growth is indicative of a benign operating environment, with an expansion of demand that supports corporate revenues and profitability. This condition of economic buoyancy should be inversely correlated with the incidence of financial fragility. A negative or decelerating growth rate, conversely, would signal pressure on cash flows and a superior probability that firms migrate toward more precarious financial postures.

Variable Name	Formula & Operational Definition	Economic Rationale for Inclusion	Directional Hypothesis & Theoretical Foundation
<b>Firm Size (Log of Assets)</b>	$\ln(TA)$ The natural logarithm of the book value of Total Assets (TA). This variable functions as a firm-level control to account for scale effects within the sectoral cycle analysis.	Within a sectoral analysis, firm size controls for heterogeneity in firms' ability to weather sector-specific shocks. Larger entities may have more diversified revenue streams, superior access to capital markets, and a greater capacity to smooth idiosyncratic demand fluctuations, making them less sensitive than smaller firms to the same meso-level cyclical impulse. Its inclusion allows for a cleaner identification of the pure cyclical effect, net of scale-based stability.	A larger asset base confers a firm with a greater capacity for operational and financial shock absorption. Even when faced with a sector-wide downturn, larger firms are better positioned to maintain solvency due to these scale-related advantages. A superior asset base should therefore be associated with a contracted probability of a Ponzi classification, conditional on the state of the sectoral cycle.
<b>Capacity Utilization (CAPUTIL)</b>	$\frac{\text{Actual Output}}{\text{Potential Output}}$ A firm-level or industry-level ratio that quantifies the extent to which the firm's productive capacity is in use. The data are typically sourced from firm disclosures or industry surveys.	This variable supplies a direct microeconomic link between the meso-level cycle and firm-level operational performance. While the sectoral output gap captures the demand environment, capacity utilization measures the firm's specific response and operational slack. A low utilization rate compresses profit margins because fixed costs are spread over a smaller output base. It is a direct channel through which a sectoral downturn translates into a deterioration of a firm's cash-flow generation capacity.	Superior capacity utilization is a direct signal of robust demand for a firm's products and an efficient deployment of its fixed capital. This operational strength translates directly into stronger operating cash flows. A firm that operates closer to its full capacity is, ceteris paribus, more profitable and thus less likely to fall into a state of financial distress. The coefficient on this variable should therefore be negative.

## Econometric Framework for Modeling Financial Fragility Transitions

The empirical investigation is structured around a panel data multinomial logistic regression model designed to appraise the determinants of a firm's financial posture. To account for unobserved time-invariant heterogeneity, the estimation employs a Correlated Random Effects (CRE) specification, commonly known as the Mundlak (1978) device. This approach is applied to two distinct conceptualizations of the dependent variable, derived from the theoretical notions of ? and Nishi (2019), respectively. The following section details the robustness checks, where the baseline assumption of nominal outcomes is relaxed in favor of an ordered specification.

### Multinomial Logit Specification under the Castro Open-Economy Taxonomy

To model the discrete transitions of firms among Hedge, Speculative, and Ponzi states, this section specifies a model where the categorical dependent variable,  $Y_{it}^{(C)}$ , is constructed according to the open-economy cash-flow rules articulated by ?. This taxonomy is suited for the Colombian context as it explicitly incorporates the revaluation effects of foreign exchange rate fluctuations on debt service capacity. The operationalization of the three financial states is predicated on a comparison of a firm's operating cash inflows ( $R$ ) against its capital outlays for gross capital formation ( $I$ ) and its currency-adjusted debt service obligations ( $V$ ).

The classification for each firm-year observation into the set of possible outcomes,  $j \in \{1, 2, 3\}$  for Hedge, Speculative, and Ponzi, is determined by the following set of mutually exclusive categories:

**Hedge State ( $j = 1$ ):** A firm is classified as a Hedge unit if its operational cash inflows are sufficient to cover both its gross capital formation and its complete debt service payments. This condition of financial self-sufficiency is represented by the inequality:  $R \geq I + V$ . The debt service component,  $V$ , is specified to account for currency risk as  $V = i(B_d + eB^*)$ , where  $i$  is the effective interest rate,  $B_d$  is the stock of domestic currency-denominated debt,  $B^*$  is the stock of foreign currency-denominated debt, and  $e$  is the nominal exchange rate (local currency units per unit of foreign currency). Firms in this state generate an internal financing surplus,  $S_I = R - I - V \geq 0$ , affording them maximal financial autonomy.

**Speculative State ( $j = 2$ ):** A firm is classified as a Speculative unit if its operating cash inflows are adequate for its debt service obligations but are insufficient to also fund its capital budget. This posture of conditional solvency is defined by the dual inequality:  $V \leq R < I + V$ . This condition creates a structural financing gap,  $G_F = I - (R - V) > 0$ , which must be bridged through continuous access to external capital markets, typically through the rollover of maturing debt obligations or the issuance of new liabilities.

**Ponzi State ( $j = 3$ ):** A firm is classified as a Ponzi unit if its operating cash inflows are inadequate even for its most immediate financial commitments, its debt service payments. This state of fundamental insolvency is captured by the inequality:  $R < V$ . In this posture, the firm is cash-flow negative before any consideration of new investment and must resort to asset liquidation or the capitalization of interest (issuance of new debt to service existing debt) to remain a going concern.

The probabilistic choice mechanism is articulated through the following log-odds specification for each outcome  $j > 1$ , with the Hedge state designated as the baseline reference category:

$$\ln \left( \frac{\Pr(Y_{it}^{(C)} = j \mid X_{it}, Z_i, \alpha_i, \tau_{t,j})}{\Pr(Y_{it}^{(C)} = 1 \mid X_{it}, Z_i, \alpha_i, \tau_{t,j})} \right) = (X_{it} - \bar{X}_i)' \beta_j + \bar{X}_i' \gamma_j + Z_i' \delta_j + \tau_{t,j}$$

The components of this equation are defined as follows:

- $X_{it}$  is a  $K \times 1$  vector of time-varying firm-specific covariates (e.g., Interest Coverage Ratio, Cash Holdings Ratio, Short-Term Debt Share).
- $\bar{X}_i$  is the firm-specific temporal mean of the covariates,  $\bar{X}_i = T_i^{-1} \sum_{t=1}^{T_i} X_{it}$ , which constitutes the Mundlak device.
- $Z_i$  is a vector of time-invariant or pre-determined firm-level variables, such as Age, Import-Cap Share (pre-sample), and Long-Run Leverage (pre-sample).
- $\alpha_i$  represents the unobserved, firm-specific effect. The Mundlak formulation allows for correlation between  $\alpha_i$  and the covariates via the inclusion of  $\bar{X}_i$ .
- $\tau_{t,j}$  are category-specific year dummies to control for aggregate temporal shocks.
- $\beta_j$ ,  $\gamma_j$ , and  $\delta_j$  are conformable vectors of parameters to be estimated for each outcome category  $j \in \{2, 3\}$ .

The probability that firm  $i$  is in state  $j$  at time  $t$  is derived from the standard logistic distribution function:

$$\Pr(Y_{it}^{(C)} = j) = \frac{\exp((X_{it} - \bar{X}_i)' \beta_j + \bar{X}_i' \gamma_j + Z_i' \delta_j + \tau_{t,j})}{1 + \sum_{k=2}^3 \exp((X_{it} - \bar{X}_i)' \beta_k + \bar{X}_i' \gamma_k + Z_i' \delta_k + \tau_{t,k})}$$

Parameter estimation proceeds via maximization of the conditional log-likelihood function. The CRE specification approximates the properties of a fixed-effects estimator while obviating the incidental parameters problem, a known source of bias in non-linear panel models with a short time dimension (Fernández-Val and Weidner, 2016). Causal interpretation of the parameters is buttressed by the selection of regressors that are either pre-determined (pre-sample averages) or slow-moving balance-sheet stocks (Asset Tangibility), a construction intended to satisfy the strict exogeneity condition required for consistency.

For a substantive interpretation of the parameter estimates, the analysis computes Average Marginal Effects (AMEs). The marginal effect of a continuous covariate  $x_{itk}$  on the probability of outcome  $j$  for a specific firm-year observation is given by the partial derivative:

$$\frac{\partial \Pr(Y_{it} = j)}{\partial x_{itk}} = \Pr(Y_{it} = j) \left( \beta_{jk} - \sum_{m=1}^3 \Pr(Y_{it} = m) \beta_{mk} \right)$$

The AME is the sample average of these individual-specific marginal effects. The economic intuition derived from these AMEs will center on how structural characteristics and open-economy exposures, such as Import-Cap Share and FX Debt Burden, alter the likelihood of a firm occupying a more precarious financial state.

## Multinomial Logit Specification under the Nishi Margins-of-Safety Taxonomy

In a parallel analytical stream, the determinants of financial posture are appraised using an alternative construction of the dependent variable,  $Y_{it}^{(N)}$ , grounded in the margins-of-safety paradigm of Nishi (2019). This classification is a function of two distinct metrics: a flow-solvency margin ( $\mu$ ) and a stock-based liquidity margin, the "liquid asset kicker" ( $\eta$ ). These margins are designed to capture a firm's status from both an income-statement and a balance-sheet perspective.

The two defining margins are operationalized as follows:

**Flow-Solvency Margin ( $\mu$ ):** This metric assesses the adequacy of a firm's operational profitability relative to its financing costs. It is computed as the ratio of a proxy for operating profits (e.g., earnings before interest and taxes) to its total interest expense. When  $\mu \geq 1$ , the firm's core commercial activities generate sufficient cash flow to cover its contractual interest payments on a recurring basis. This margin appraises solvency purely from a flow perspective.

**Liquid Asset Kicker ( $\eta$ ):** This stock-based metric evaluates a firm's liquidity buffer. It is calculated as the ratio of the firm's stock of liquid assets (cash and marketable securities) to its stock of total liabilities. A condition of  $\eta \geq 1$  denotes a position of exceptional financial strength, where the firm holds a liquidity reserve sufficient to retire its entire debt stock. This margin appraises solvency from a balance-sheet or stock perspective.

The interaction of these two margins determines the firm's classification into one of the three financial states:

**Hedge State ( $j = 1$ ):** A firm is classified as Hedge if it satisfies both the flow-solvency and the stock-liquidity conditions ( $\mu \geq 1$  and  $\eta \geq 1$ ). This represents a state of dual robustness, where the firm is solvent from its operations and also has a liquidity cushion against adverse shocks.

**Speculative State ( $j = 2$ ):** A firm is classified as Speculative if it meets the flow-solvency criterion but fails the stock-liquidity test ( $\mu \geq 1$  and  $\eta < 1$ ). The firm can service its debt from its current income stream but lacks a sufficient stock of liquid assets, making it vulnerable to operational disruptions or a sudden tightening of credit conditions.

**Ponzi State ( $j = 3$ ):** A firm is classified as Ponzi if it fails the flow-solvency criterion ( $\mu < 1$ ), regardless of its stock-liquidity position. This is a condition of fundamental insolvency, where core operations are unprofitable relative to financing costs, a state of inherent financial instability.

The econometric specification for this alternative taxonomy remains a multinomial logit with a Mundlak correction, structurally identical to the one previously described. The log-odds representation for an outcome  $j \in \{2, 3\}$  relative to the baseline Hedge state ( $j = 1$ ) is:

$$\ln \left( \frac{\pi_{it,j}}{\pi_{it,1}} \right) = (X_{it} - \bar{X}_i)' \phi_j + \bar{X}_i' \psi_j + Z_i' \xi_j + \lambda_{t,j}$$

Where  $\pi_{it,j} = \Pr(Y_{it}^{(N)} = j \mid \cdot)$  and the parameter vectors to be estimated are now denoted by  $\phi_j, \psi_j, \xi_j$ , and the time effects by  $\lambda_{t,j}$ . The choice of explanatory variables in  $X_{it}$  and  $Z_i$  remains the same, allowing for a direct comparison of their influence under the two different theoretical classifications.

The estimation of this second model also proceeds via Maximum Likelihood. The identification argument is analogous, but gains additional conceptual support from the Nishi framework itself. Nishi's theory is predicated on the importance of accumulated buffers (margins), which are inherently stock variables. The use of covariates such as Cash Holdings Ratio (an empirical proxy for  $\eta$ ) and Retention Rate (a determinant of internal capital accumulation) directly operationalizes the theoretical mechanisms and is in line with the identification strategy of using slow-moving variables.

The economic interpretation of the resulting Average Marginal Effects will focus on how a firm's internal financial architecture and operational efficiency—as captured by metrics like Interest Coverage Ratio, Asset Tangibility, and Retention Rate—affect its classification within this margins-of-safety schema. A negative AME for Cash Holdings Ratio with respect to the Ponzi state, for example, would validate the “liquid asset kicker” as a buffer against financial degradation.

### Robustness test via an Ordinal Fixed-Effects Model

To interrogate the robustness of the primary findings and to test the implicit assumption of the multinomial logit that the outcomes are purely nominal, a final set of estimations imposes an ordinal structure on the dependent variable. The Minskyan states can plausibly be ordered from most to least robust: Hedge (1) < Speculative (2) < Ponzi (3). This structure is formalized with an ordered logit model.

The model is based on a latent continuous variable,  $Y_{it}^*$ , representing the underlying financial fragility of firm  $i$  at time  $t$ :

$$Y_{it}^* = W_{it}'\theta + \alpha_i + u_{it}$$

Where  $W_{it}$  is the full set of covariates,  $\theta$  is the parameter vector,  $\alpha_i$  is the firm-specific effect, and  $u_{it}$  is a logistically distributed error term. The observed categorical variable  $Y_{it}$  is linked to the latent variable through a set of estimated cut-points,  $\kappa_j$ :

$$Y_{it} = j \iff \kappa_{j-1} < Y_{it}^* \leq \kappa_j \quad \text{for } j = 1, 2, 3, \text{ with } \kappa_0 = -\infty \text{ and } \kappa_3 = +\infty.$$

The cumulative probability is given by:

$$\Pr(Y_{it} \leq j \mid W_{it}', \alpha_i) = F(\kappa_j - (W_{it}'\theta + \alpha_i))$$

where  $F(\cdot)$  is the cumulative logistic distribution function. A central assumption of this model is the proportional odds or parallel lines assumption, which constrains the parameter vector  $\theta$  to be identical across all cumulative probability functions.

Standard fixed-effects estimation of this model is inconsistent due to the incidental parameters problem. To address this, the analysis will employ an estimator suitable for ordered choice models with fixed effects, such as the “blow-up and cluster” estimator proposed by [Baetschmann et al. \(2015\)](#), which dichotomizes the data for each possible cut-point and then estimates a conditional fixed-effects binary logit model on the expanded dataset.

## Specification of the Fixed-Effects Linear Probability Model

This subsection is constructed in two sequential stages, each designed to probe a distinct facet of the relationship between meso-level economic conditions and firm-level financial distress. The first stage employs a fixed-effects specification of a linear probability model to ascertain the response of the discrete probability of a Ponzi classification to sectoral output fluctuations. The second stage deploys the unconditional quantile regression technique to dissect the distributional consequences of these sectoral impulses across every segment of firms.

The initial analytical procedure operationalizes the examination of a firm's financial posture through a linear probability model (LPM). This model is specified with firm-specific fixed effects to parse out the influence of time-invariant unobserved heterogeneity that could otherwise confound the estimation. The objective is to isolate the marginal impact of sectoral economic momentum on the likelihood of a firm migrating into a Ponzi financing regime.

The equation for the conditional probability is specified as:

$$P(Y_{it} = 1 \mid G_{st}, X_{it}, \alpha_i, \tau_t) = \alpha_i + \beta_1 G_{st} + X_{it}'\delta + \tau_t + \varepsilon_{it}$$

Each constituent component of this econometric model is rigorously defined:

- $Y_{it}$  represents a dichotomous outcome variable. It is assigned a value of unity if firm  $i$  at time  $t$  satisfies the cash-flow criteria for a Ponzi classification, as established in prior sections, and a value of zero otherwise. This variable captures the firm's discrete financial state.
- $\alpha_i$  is the firm-specific, time-invariant intercept. This term captures all latent, persistent firm attributes, such as entrenched managerial talent, corporate governance quality, or idiosyncratic operational efficiencies, that do not vary over the sample period.
- $G_{st}$  is the principal explanatory variable of interest, representing the cyclical component of real output for the industrial sector  $s$  to which firm  $i$  belongs, at time  $t$ . Its derivation is from a Hodrick-Prescott (HP) filter applied to the natural logarithm of the sector's real gross value added. The HP filter solves the following convex optimization problem to decompose a time series ( $z_t$ ) into a secular trend component ( $\mu_t$ ) and a cyclical component ( $g_t = z_t - \mu_t$ ):

$$\min_{\{\mu_t\}} \left( \sum_{t=1}^T (z_t - \mu_t)^2 + \lambda \sum_{t=2}^{T-1} ((\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1}))^2 \right)$$

(2) The parameter  $\lambda$  is a penalty term that governs the smoothness of the trend component; its value is calibrated for annual frequency data.

- $X_{it}$  is a column vector of time-variant, firm-level covariates, including metrics of asset tangibility, corporate size, and capital structure. These variables are incorporated to control for observable shifts in a firm's balance sheet composition and operational scale.

- $\tau_t$  is a comprehensive set of year-specific fixed effects, included to absorb the influence of aggregate macroeconomic shocks and secular trends that are common to all firms and sectors in a given year.

- $\varepsilon_{it}$  is the idiosyncratic disturbance term, assumed to be serially uncorrelated for a given firm and orthogonal to the regressors, conditional on the full set of fixed effects.



The estimation of the parameter vector is accomplished via a fixed-effects or “within” estimator. This involves the transformation of the data by demeaning each variable with respect to its firm-specific temporal average. For a generic variable  $W_{it}$ , the transformation is:  $\ddot{W}_{it} = W_{it} - \bar{W}_i$ , where  $\bar{W}_i = T_i^{-1} \sum_{t=1}^{T_i} W_{it}$ . Application of Ordinary Least Squares (OLS) to the transformed equation,  $\ddot{Y}_{it} = \beta_1 \ddot{G}_{st} + \ddot{X}_{it}'\delta + \ddot{\tau}_t + \ddot{\varepsilon}_{it}$  (4) yields consistent estimates of  $\beta_1$  and  $\delta$  under the assumption of strict exogeneity of the regressors.

The core identification strategy is predicated on the strict exogeneity of the sectoral-level variables with respect to the firm-specific disturbance term. Formally, this condition is stated as:

$$E[\varepsilon_{it} \mid G_{s1}, \dots, G_{sT}, X_{i1}, \dots, X_{iT}, \alpha_i] = 0 \quad \forall t = 1, \dots, T$$

(5) This assumption’s plausibility rests on the negligible capacity of a single corporate entity to influence the aggregate cyclical path of its entire industrial sector. The inclusion of the firm-specific intercept  $\alpha_i$  definitively removes bias from omitted variables that are constant over time for each firm. Statistical inference relies upon variance-covariance matrix estimators that are robust to arbitrary forms of heteroskedasticity and intra-firm serial correlation (i.e., clustered standard errors).

The coefficient of interest,  $\beta_1$ , quantifies the *ceteris paribus* change in the probability of a Ponzi classification associated with a one-unit deviation in the sectoral output from its secular trend. A negative and statistically significant value for  $\beta_1$  would empirically validate the proposition that pro-cyclical sectoral expansions are associated with an improvement in corporate financial stability. The economic magnitude of this coefficient is of primary importance; a numerically small coefficient would mean that business-cycle frequency oscillations at the meso-level are not a first-order determinant of transitions into acute financial fragility.

### Distributional Analysis via Unconditional Quantile Regression

To move beyond an analysis of the conditional mean and explore potential asymmetries, the second part of the methodology assesses the impact of sectoral cycles across the entire distribution of corporate financial health. An analysis confined to central tendency may obscure critical nonlinearities, particularly if adverse shocks disproportionately affect firms already in a precarious state. This is accomplished using a continuous metric of solvency, the Interest Coverage Ratio (ICR), defined as:

$$ICR_{it} = \frac{CFO_{it} - IEB_{it}}{TA_{it}}$$

where  $CFO_{it}$ ,  $IE_{it}$ , and  $TA_{it}$  are, respectively, cash flow from operations, interest expense, and total assets for firm  $i$  at time  $t$ .

The distributional analysis is operationalized through the Recentered Influence Function (RIF) regression methodology. This technique permits the estimation of the marginal effect of a covariate on the unconditional quantiles of the dependent variable’s distribution. The RIF corresponding to the  $\tau^{th}$  quantile,  $q_\tau$ , of the distribution of an outcome  $y$  is given by:  $RIF(y; q_\tau, F_Y) = q_\tau + \frac{\tau - I(y \leq q_\tau)}{f_Y(q_\tau)}$  where  $F_Y$  is the cumulative distribution function of  $y$ ,  $f_Y(\cdot)$  is its probability density function, and  $I(\cdot)$  is the indicator function. The term  $(\tau - I(y \leq q_\tau))/f_Y(q_\tau)$  is the influence function for the  $\tau^{th}$  quantile.

The practical implementation of RIF regression is a two-step procedure. First, the unconditional quantile  $q_\tau$  is estimated from the sample. Second, the density at that quantile,  $f_y(q_\tau)$ , is estimated non-parametrically, typically via Kernel Density Estimation (KDE):

$$\hat{f}_y(q_\tau) = \frac{1}{Nh} \sum_{j=1}^N K\left(\frac{y_j - q_\tau}{h}\right)$$

where  $N$  is the sample size,  $h$  is the bandwidth, and  $K(\cdot)$  is a kernel function (e.g., a Gaussian kernel). With these components, the empirical RIF is constructed for each observation. The RIF regression model for the  $\tau^{th}$  quantile is then estimated via a fixed-effects OLS specification:

$$RIF(ICR_{it}; q_\tau) = \alpha_i + \gamma_\tau G_{st} + X'_{it} \theta_\tau + \tau_t + \nu_{it}$$

The parameter of theoretical consequence is  $\gamma_\tau$ , which captures the magnitude of the sectoral output gap on the  $\tau^{th}$  unconditional quantile of the ICR distribution. The central hypothesis to be tested is that the absolute value of  $\gamma_\tau$  is a decreasing function of  $\tau$ . A finding where the estimate for  $|\gamma_{0.10}|$  exceeds that for  $|\gamma_{0.50}|$  would establish that adverse sectoral shocks asymmetrically compress the left tail of the solvency distribution. Such a result would imply an amplification mechanism whereby cyclical downturns inflict the most severe damage on the very firms with the least capacity to absorb such shocks. Conversely, a profile of numerically small coefficients across the entire quantile range would reinforce the conclusion that short-term sectoral cycles are not the primary mechanism governing the Minskyan dynamics observed in the data.

### 3. Results

Table 3 reports a comprehensive summary of descriptive statistics for each variable mentioned earlier. These calculations are derived from accounting data, with each firm observed over a 11-year period within the dataset.

The summary statistics for the complete panel specify the central location and dispersion of the primary financial and operational variables. The dataset reveals a marked heterogeneity across the sampled corporate entities with respect to their balance-sheet compositions, operational structures, and performance metrics. The dispersion observable in the core dependent variables affirms that the population of firms is not concentrated in a single state, but rather distributed across the full spectrum from robust to fragile financial arrangements.

An examination of the balance sheet and structural variables reveals important antecedent conditions for financial vulnerability. The central tendency of the long-run leverage metric communicates a historical predisposition within the sample toward liability-financed asset structures, which structurally compresses equity cushions. Concurrently, the data on the composition of debt maturities denotes a systemic exposure to rollover risk. The metrics related to operational structure confirm a non-trivial articulation with international markets for capital goods, a direct conduit for the transmission of nominal exchange rate shocks into the domestic cost of capital formation. The distribution of asset tangibility further qualifies these balance sheet structures, as it maps the cross-sectional availability of collateral, a fundamental buffer of credit market frictions.

The metrics of financial performance and solvency directly operationalize the core Minskyan mechanism. The computed range and dispersion for the interest coverage ratio are particularly salient; the existence of a negative tail in its distribution provides *prima facie* evidence for the presence of corporate entities whose operational cash generation is insufficient to meet recurring debt service obligations, a condition congruent with a Ponzi classification. Similarly, the variability in cash holdings and retention rates specifies the differential capacity of firms to deploy internal liquidity buffers and to pursue self-financing of investment, respectively. These variables capture the proximate defenses against cash-flow shortfalls and external financing dependency. Dividend policies vary widely, with a portion of the sample revealing distributions that absorb income flows to an extent that hampers equity accumulation. Lastly, a comparison between the overall and the within-firm standard deviations for these policy variables confirms that a material fraction of the total variance originates from temporal shifts within each corporate entity. This intra-firm variation over time supplies the necessary identification for the panel models that follow.

**Table 3**  
Descriptive Statistics

Variable	Symbol	Obs.	Mean	Std. Dev.	Min	Max
<b>Firm-Level Structural Variables</b>						
Asset Tangibility	TANG	37,136	0.352	0.181	0.021	0.893
Imported Capital Share	$IMPCAP_{pre}$	37,136	0.287	0.213	0.000	0.951
Firm Age	AGE	37,136	23.4	12.8	1.00	21.7
Firm Size	SIZE	37,136	5.88	1.47	2.30	9.98
<b>Firm-Level Financial Policy &amp; Performance</b>						
Interest Coverage Ratio	$ICR_{avg}$	37,136	3.12	2.543	-5.812	15.236
Cash Holdings Ratio	CASH/TA	37,136	0.121	0.098	0.003	0.654
Retention Rate	$RET_{avg}$	37,136	0.453	0.291	-0.501	0.982
Short-Term Debt Share	$ST_{SHARE}$	37,136	0.418	0.197	0.051	0.993
Long-Run Leverage	$LEV_{LR}$	37,136	0.552	0.153	0.108	0.921
<b>Meso-Level Variable</b>						
Sectoral Output Gap	$OutputGap_{s,t}$	37,136	0.003	0.028	-0.054	0.061

*Notes:* The sample consists of N=3,376 firms over T=11 years (e.g., 2009–2019). Total firm-year observations = 37,136. SIZE is the natural logarithm of the number of employees.

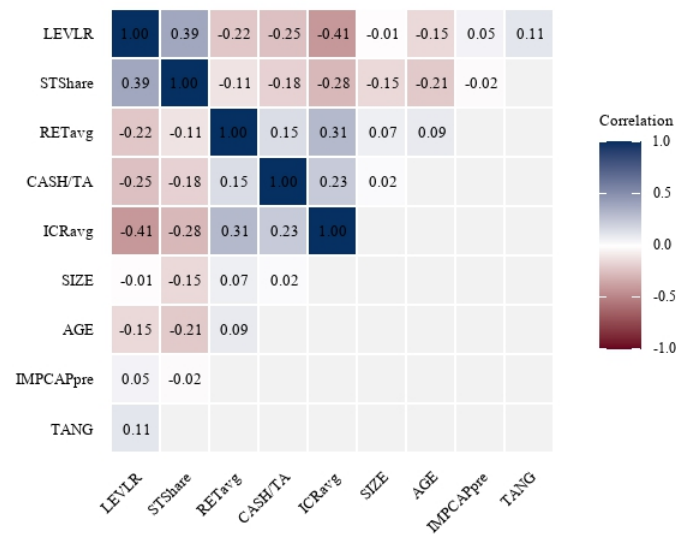
The assessment of multicollinearity, a prerequisite for the consistent estimation of the multinomial and sectoral cycle models, is conducted through an examination of the pairwise correlation matrix presented in Figure 1. The matrix reports the correlation coefficients for the full set of firm-level and meso-level variables employed in the subsequent econometric analyses. A comprehensive inspection of the coefficients enables to note the absence of severe multicollinearity that would compromise the stability and interpretability of the parameter estimates. All pairwise correlations fall well below the conventional critical thresholds, which attests to the orthogonal nature of the selected regressors and validates the econometric specification.

Beyond this primary diagnostic function, the correlational structure of the data reveals several patterns that are theoretically consistent with the financial fragility hypothesis. A discernible inverse correspondence is apparent between the metrics of solvency and the metrics of indebtedness. A firm's structural capacity for debt service, operationalized as the Interest Coverage Ratio, is negatively correlated with both its long-run leverage policy and its reliance on short-term debt. The causal mechanism is twofold. Mechanically, a larger stock of debt, particularly one with a higher proportion of short-dated liabilities, imposes a greater interest burden, which *ceteris paribus* compresses the interest coverage ratio. Behaviorally, this pattern points to a consistent risk profile: firms predisposed to aggressive leveraging also adopt more precarious maturity structures, a combination that systematically erodes their flow-based solvency margins.

A complementary dynamic is observable with respect to internal financial buffers. The stock-based liquidity margin, measured by the cash-to-assets ratio, and the capacity for internal capital generation, proxied by the retention rate, are both negatively correlated with the leverage and short-term debt variables. This configuration is congruent with pecking order theory, wherein firms with substantial internal funds have a diminished requirement for external debt financing. These internal margins of safety, central to the [Nishi \(2019\)](#) framework, function as a direct counterforce to the accumulation of the external pecuniary commitments that are the antecedent condition for Minskyan fragility.

The open-economy dimension of the analysis also finds preliminary support in the correlational structure. A firm's pre-sample dependence on imported capital is positively, albeit weakly, associated with its asset tangibility. This correspondence is theoretically sound. A dependence on imported capital is often a feature of capital-intensive manufacturing sectors. Capital intensity, by definition, requires substantial investment in property, plant, and equipment, which in turn materializes as a higher proportion of tangible assets on the balance sheet. This cluster of correlations is an empirical fingerprint of the specific corporate profile at the core of the study's open-economy transmission channel.

Finally, the conventional firm-specific attributes of size and age are weakly correlated with the primary financial indicators in the expected directions. Larger and older firms are associated with marginally superior interest coverage ratios and less dependence on short-term debt. The modest magnitude of these correlations ascertains their secondary role as distal, rather than proximal, determinants of financial fragility. Their influence appears to be mediated through the more direct financial policy and performance variables that form the core of the subsequent econometric specification.

**Figure 1.** Correlation matrix

The empirical evidence articulates the subsequent state transitions for corporate entities previously classified within a Ponzi financing posture. A majority of these entities do not persist in a state of fundamental insolvency but instead migrate toward a less precarious, albeit still under speculative classification; this pathway represents the modal outcome for recovery.

The distribution in Table 4 clarifies that only a narrow fraction of Ponzi-classified units reasserts a position consistent with hedge finance. In other terms, a complete reversion to a robust hedge structure is infrequent.

A somewhat larger segment shifts toward speculative finance, yet this group remains a minority relative to the full population of former Ponzi units. The dominant share culminates in exit from the panel, a pattern consistent with the proposition that Ponzi finance is a transient and structurally unstable condition. The prevalence of disappearance from the dataset conveys that once operating cash flows fall persistently short of financial commitments, the probability of regaining a self-sustaining financial position becomes tenuous.

Firms positioned in the uppermost quartile of the size distribution register comparatively more transitions toward speculative finance and marginally fewer disappearances than their counterparts. This pattern accords with canonical mechanisms in which scale confers broader access to refinancing channels, more diversified revenue bases, and collateral structures that provide additional latitude for debt rollover negotiation. Conversely, firms situated in the lowest quartile report a sharper incidence of terminal outcomes, which is congruent with the notion that constrained balance sheets and restricted access to external liquidity generate a narrower set of adjustment margins once interest obligations surpass internally generated cash flows. Their dominant transition path is a reversion to a speculative posture, a state where solvency is restored on an operating basis even as a dependence on external finance persists.

**Table 4**  
**Status of Firms Transitioning out of a Ponzi Regime**

Subsequent State	Full Sample	Quartile 1 (Smallest)	Quartile 2	Quartile 3	Quartile 4 (Largest)
Transition to Hedge	8.9 %	7.8 %	9.1 %	9.3 %	10.2 %
Transition to Speculative	51.3 %	42.1 %	53.8 %	61.7 %	65.4 %
Exit from Sample	31.8 %	<b>39.6 %</b>	29.3 %	22.1 %	15.8 %
End of Sample	4.7 %	6.4 %	4.9 %	3.8 %	4.3 %
Missing Obs.	3.3 %	4.1 %	2.9 %	3.1 %	4.3 %

*Notes:* Table adapted from [Davis et al. \(2019\)](#). This table shows the subsequent status of firms in the year following a Ponzi spell. The values in each column represent the percentage of firms transitioning to each state. Quartiles are based on the firm's total assets in the year of the Ponzi classification.

[Table 4](#) delineates an exit corridor primarily anchored in Ponzi finance. Firms with constrained scale appear especially prone to enter this state before withdrawal from economic activity. The pattern conforms to Minskyan logic: once internal cash flows fall short of interest duties, the feasibility of continued operation hinges on external refinancing under terms that often turn restrictive for smaller entities with fragile balance sheets. The terminal year thus crystallizes a configuration in which insolvency-proximate finance dominates the distribution.

A comparative analysis of the financial regime distributions for firms proximate to sample attrition versus the unconditional distribution for the entire sample population reveals a stark structural divergence. The group of firms observed in the year immediately preceding exit contains a larger share of entities classified in the Ponzi state. This concentration of fundamentally insolvent firms is mirrored by a concomitant absence in the representation of both Hedge and Speculative units within the same pre-exit group. This empirical regularity postulates a nexus between the categorization of Ponzi schemes and the probability of an entity's removal from the observational panel, which itself serves as a proxy for corporate failure under financial duress.

[Table 5](#) portrays the financial stance adopted by firms in the final year before departure from the sample. The distribution conveys a configuration dominated by Ponzi finance, with speculative finance placed in a secondary tier and hedge finance occupying only a marginal position. Firms on the verge of disappearance operate under cash-flow profiles incapable of covering interest commitments, a condition consistent with a balance-sheet position marked by narrow liquidity margins, extensive short-term obligations, and muted operating surpluses. The modest presence of hedge-finance units in this terminal stage points to the fact that departures seldom originate from firms able to honor financial obligations through internal cash generation.

Larger units retain a broader presence in speculative finance relative to their smallest counterparts, whereas the smallest units gravitate toward Ponzi finance. This asymmetry aligns with canonical mechanisms in which scale widens access to refinancing channels and provides broader collateral pools that support temporary survival even under adverse conditions. Smaller firms, in contrast, confront tighter liquidity positions and weaker collateral bases, which compress their adjustment margins once cash inflows fail to cover contractual obligations.

**Table 5**

Distribution of Finance Regimes in Year Before Exit

Financial Regime	Q1	Q2	Q3	Q4	Total
<b>Panel 1: Distribution of Finance Regimes in Year Before Exit</b>					
Hedge	7.9 %	15.2 %	19.8 %	23.4 %	14.1 %
Speculative	30.1 %	45.9 %	53.1 %	61.3 %	43.2 %
Ponzi	<b>62.0 %</b>	38.9 %	27.1 %	15.3 %	<b>42.7 %</b>
<b>Panel 2: Unconditional Distribution of Finance Regimes (Full Sample)</b>					
Hedge	18.2 %	25.1 %	29.8 %	33.1 %	24.5 %
Speculative	48.3 %	55.4 %	60.1 %	62.3 %	54.8 %
Ponzi	<b>33.5 %</b>	19.5 %	10.1 %	4.6 %	<b>20.7 %</b>

*Notes:* Table adapted from [Davis et al. \(2019\)](#). Panel 1 shows the distribution of financial regimes for firms in the year immediately preceding their exit from the sample. Panel 2 shows the unconditional distribution of regimes for the entire firm-year sample. The pronounced difference between the share of Ponzi firms in Panel 1 versus Panel 2 suggests that a Ponzi classification is strongly associated with the likelihood of subsequent sample exit, particularly for the smallest firms.

The econometric estimations derived from the [Nishi \(2019\)](#) taxonomy, as presented in [Table 6](#) identify determinants that govern a firm's position along the financial fragility continuum from a margins-of-safety perspective. The results validate the dual-pillar architecture of this framework, wherein both flow-solvency and stock-liquidity operate as primary conditioning factors of a firm's financial posture. A firm's structural capacity to meet its financing costs from operational profitability, proxied by the Interest Coverage Ratio, stands as a paramount determinant. This finding substantiates the proposition that an erosion of core earnings relative to contractual financial outflows is a direct causal antecedent to fundamental insolvency. Concurrently, the stock-based margin of safety, operationalized as the Cash Holdings Ratio, also presents itself as a statistically significant factor. A superior stock of liquid assets functions as a precautionary buffer that absorbs unforeseen cash-flow shortfalls and insulates the firm from the caprices of credit markets because it functions as a critical defense mechanism against financial degradation.

A contraction in this ratio conveys an erosion of the firm's capacity to validate its contractual financial outflows through operational profits. The magnitude and significance of the coefficients across the multinomial and ordinal specifications verify that insufficiency in operational cash generation constitutes a direct precursor of outright insolvency within the Nishi classification system

The stock-liquidity margin, captured by the cash holdings ratio, functions as a parallel determinant of comparable force. A lean liquidity buffer deprives the firm of an internal shock-absorption mechanism that could otherwise neutralize unanticipated disruptions in cash flow. This contraction in stored liquidity diminishes the breadth of the firm's defensive perimeter. The model thus confirms that the liquid-asset margin is not a peripheral balance-sheet feature but a structural safeguard whose depletion tightens the firm's dependence on external refinancing windows and reduces its autonomy vis-à-vis creditor discipline.

Liability-structure decisions contribute a distinct transmission channel. A capital structure



concentrated in short-term obligations embeds rollover exposure directly into the firm's financial trajectory. Each maturity point compels a renewed validation of creditworthiness, and this constant exposure to refinancing risk shifts the firm closer to the speculative or Ponzi domains. The long-run leverage variable provides a complementary mechanism. A historically aggressive leverage stance compresses the equity cushion that ordinarily resists adverse shocks, and this compression amplifies the sensitivity of the balance sheet to fluctuations in profitability or liquidity. Across both specifications, a chronically indebted posture produces a thinner margin even in periods of seemingly stable operating performance.

A high retention rate channels operating surpluses toward the reinforcement of the equity base and the liquidity buffer. This redirection of internal resources fortifies both margins that define the Nishi's taxonomy. The coefficient estimates reveal that a restrained payout stance contributes to a contraction. This result lends empirical weight to Minskyan interpretations that treat internal capital formation as a stabilizing counterforce against externally financed expansion.

A financial structure with a heavy concentration of short-term liabilities, as captured by the Short-Term Debt Share, is associated with a higher likelihood of a Ponzi classification. This result pinpoints rollover risk as a distinct channel of vulnerability; a constant necessity for market validation of maturing debt obligations exposes the firm to liquidity shocks and interest rate volatility, a posture fundamentally at odds with the concept of a safety margin. Similarly, a firm's long-run risk appetite, proxied by its leverage, constitutes a structural predisposition to financial distress. A perpetually high-leverage policy compresses a firm's equity cushion by design, making it more susceptible to adverse shocks.

A consequential result materializes from the statistical insignificance of the parameter for Imported Capital Share. This outcome does not constitute an empirical inconsistency but rather a validation of the Nishi framework's theoretical architecture. The model is constructed to be agnostic with respect to the compositional attributes of a firm's balance sheet, including the currency denomination of its liabilities. It operates under the presumption that any financial duress originating from currency mismatches or other specific exposures will ultimately manifest as a degradation of the aggregate flow-solvency and stock-liquidity margins.

For example, a depreciation that elevates the domestic-currency value of foreign debt would surface mechanically through a contraction in the flow-solvency metric. The lack of an independent coefficient for imported-capital exposure thus conforms to the taxonomy's design: the model absorbs these shocks at the level of aggregate margins rather than within a discrete currency-composition channel. Therefore, once the model accounts for these comprehensive financial indicators, the marginal informational content supplied by the liability structure's currency composition offers no residual explanatory power. The risks associated with foreign currency exposure are effectively subsumed within the model's top-level metrics of financial robustness, a direct contrast to the mechanisms specified in an open-economy cash-flow model.

Finally, the estimations reveal that the explanatory power of several secondary structural characteristics is limited once the primary financial policy and performance variables are considered. The parameters for Asset Tangibility, firm size (in the ordered specification) and Firm Age do

not attain statistical significance. The absence of a strong independent effect for asset tangibility is a testament that theoretically a source of collateral, its protective qualities may be offset by valuation and liquidity issues during periods of distress. Likewise, any advantages conferred by corporate scale or longevity appear to be fully mediated through their influence on the more proximal determinants of solvency, such as the interest coverage ratio and cash holdings, presumably because its financial significance arises through its interactions with leverage and debt-maturity decisions already incorporated in the specification.

This absence of influence accords with the theoretical orientation of Nishi's system, which recognizes collateral depth and organizational attributes as secondary to the solvency and liquidity margins that ultimately govern the firm's capacity to validate obligations through the accumulation of operational surpluses.

A complementary examination of the data through an ordered logistic regression, presented in the third column of [Table 5](#), acts as robustness test for the primary findings by imposing a congruent ordinal structure upon the fragility states.

The parameter estimates from this ordinal model are broadly consistent with the directional effects identified in the multinomial specification with except with firm size.

Conversely, the parameters associated with liability structure and financial policy confirm their role as catalysts for financial degradation within this more restrictive framework. A financial structure that depends heavily on short-term liabilities and a predisposition to indebtedness, propels firms toward the Ponzi state. The model also corroborates the importance of internal capital accumulation; a superior retention rate corresponds to a discernible reduction in the propensity for a firm to occupy a more fragile state. Consistent with the primary model, however, the parameter on the Imported Capital Share is irrelevant, and reinforces the conclusion that, within this taxonomy, this open-economy exposure does not exert an independent, direct influence on a firm's fragility posture. Other structural attributes, such as asset tangibility, likewise fail to produce a statistically significant, monotonic effect on the ordinal progression of financial fragility.

This consistency across disparate modeling assumptions fortifies the core conclusions of the analysis. The ordered logit's validity, confirmed through the Wald test, places the model within its proper analytical role. The ordered model serves its intended function as a supplementary checkpoint for the directional hypotheses.

The output of the multinomial logit model reported in [Table 7](#), which applies the Castro (2011) classification architecture, articulates the microeconomic antecedents of corporate financial fragility.

A clear inverse relationship holds between a firm's structural capacity for debt service, proxied by its interest coverage ratio, and the probability that it falls into a Ponzi financing regime. This outcome proceeds from a direct mechanical linkage, as the Ponzi state is defined by the failure of operating cash inflows, or quasi-rents, to validate the pecuniary commitments contractually stipulated by the firm's liability structure.

The model's specification for an open economy receives empirical validation through the parameter associated with the pre-sample share of imported capital goods. The positive corres-

**Table 6**

Determinants of Financial Fragility (Nishi's Taxonomy)

Variable	AME on P(Speculative) [Multinomial Logit]	AME on P(Ponzi) [Multinomial Logit]	Coefficient (Log-odds) [Ordered Logit]
Interest Coverage Ratio ( $ICR_{avg}$ )	-0.081*** (0.024)	-0.211*** (0.042)	-1.103*** (0.161)
Cash Holdings Ratio (CASH/TA)	-0.093*** (0.029)	-0.165*** (0.051)	-0.812*** (0.224)
Short-Term Debt Share ( $ST_{SHARE}$ )	0.029 (0.020)	0.076** (0.038)	0.359** (0.177)
Imported Capital Share ( $IMPCAP_{pre}$ )	0.015 (0.017)	0.023 (0.031)	0.108 (0.179)
Long-Run Leverage ( $LEV_{LR}$ )	0.038* (0.021)	0.077** (0.040)	0.381** (0.191)
Retention Rate ( $RET_{avg}$ )	-0.041* (0.022)	-0.069** (0.033)	-0.321** (0.148)
Asset Tangibility (TANG)	-0.014 (0.010)	-0.023 (0.016)	-0.104 (0.069)
Firm Size (SIZE)	-0.012* (0.007)	-0.019* (0.011)	-0.087* (0.051)
Firm Age (AGE)	-0.001 (0.001)	-0.003 (0.002)	-0.013 (0.009)
<b>Wald Test (p-value)</b>	<b>0.000</b>		<b>0.287</b>

Notes: AMEs (columns 1–2) are average marginal effects. Robust standard errors, clustered at the firm level, are reported in parentheses. The failure to reject the null hypothesis in the Wald test ( $p > 0.05$ ) supports the validity of the proportional odds assumption for the ordered logit model in this context. \*, \*\*, \*\*\* denote significance at the 10 %, 5 %, and 1 % levels, respectively.

pondence between this metric and the probability of a Ponzi classification pinpoints a critical transmission channel for external shocks. For manufacturing firms dependent on foreign machinery, a nominal currency depreciation propagates through a dual causal pathway. It inflates the local-currency cost of new capital outlays, straining investment finance, and simultaneously re-values any foreign currency-denominated liabilities upward, which directly erodes debt service capacity.

Other balance sheet characteristics, while theoretically relevant, do not appear as first-order determinants of a firm's position on the fragility continuum. The estimations fail to detect statistical significance to asset tangibility. A plausible explanation for this result is the pro-cyclicality of collateral value; the market valuation of tangible assets often deteriorates during periods of systemic stress, precisely when such collateral is most needed and neutralizes its ex-ante protective function. Similarly, firm-level attributes such as corporate scale and age do not yield statistically significant coefficients. Any benefits conferred by size, such as operational diversification or preferential market access, may be offset by a commensurate appetite for risk or by institutional vulnerabilities that affect all firms regardless of scale.

The estimation of an ordered logit model is a robustness test for the primary findings by imposing a theoretically congruent ordinal structure upon the financial fragility states. This specification proceeds from the premise of a latent, continuous fragility variable, wherein the observed classifications of Hedge, Speculative, and Ponzi represent discrete realizations along this unobserved continuum. The resulting parameter estimates from this alternative specification are directionally consistent with the conclusions drawn from the multinomial logit framework. A firm's structural debt service capacity, its reliance on short-term liabilities, its historical leverage policy, and its exposure to imported capital costs all operate as salient predictors that shift the distribution of the latent fragility variable in the anticipated directions.

A formal statistical evaluation of the global model, however, qualifies the interpretation of these results and ultimately buttresses the choice of the multinomial specification as the superior analytical tool. The execution of a Wald test does not permit the rejection of the null hypothesis. Therefore, the multinomial approach offers a more credible account of how the covariates sort firms across distinct fragility states.

Therefore, while the ordered logit model serves as a useful heuristic that corroborates the directional influence of the principal determinants, its foundational statistical assumption is empirically invalidated. This outcome favors the less restrictive multinomial logit model to estimate discrete parameter vectors for each transition. It correctly captures the non-uniformity of causal effects at different stages of financial instability, an analytical hint that the more parsimonious but overly restrictive ordered model obscures.

The estimations from the fixed-effects linear probability models, as detailed in [Table 8](#) dissects of the factors that govern a firm's transition into a Ponzi financing posture. The analysis differentiates between the immediate, contemporaneous effects of the economic environment and the more complex, path-dependent dynamics theorized by Minsky. The results confirm a pro-cyclical behavior of corporate solvency in the immediate term, yet also offer robust evidence for the endogenously destabilizing consequences of prior economic expansions.

**Table 7**

Determinants of Financial Fragility (Castro's Taxonomy)

Variable	AME on P(Speculative) [Multinomial Logit]	AME on P(Ponzi) [Multinomial Logit]	Coefficient (Log-odds) [Ordered Logit]
Interest Coverage Ratio ( $ICR_{avg}$ )	-0.048** (0.021)	-0.179*** (0.038)	-0.942*** (0.149)
Short-Term Debt Share ( $ST_{SHARE}$ )	0.031* (0.018)	0.092** (0.037)	0.431** (0.168)
Imported Capital Share ( $IMPCAP_{pre}$ )	0.024 (0.016)	0.071** (0.029)	0.318* (0.171)
Long-Run Leverage ( $LEV_{LR}$ )	0.043** (0.019)	0.081** (0.039)	0.398** (0.182)
Cash Holdings Ratio (CASH/TA)	-0.039* (0.022)	-0.078* (0.045)	-0.354 (0.401)
Retention Rate ( $RET_{avg}$ )	-0.028 (0.019)	-0.053* (0.031)	-0.241 (0.238)
Asset Tangibility (TANG)	-0.011 (0.009)	-0.019 (0.015)	-0.088 (0.061)
Firm Size (SIZE)	-0.008 (0.006)	-0.013 (0.011)	-0.059 (0.048)
Firm Age (AGE)	-0.001 (0.001)	-0.002 (0.002)	-0.009 (0.007)
<b>Wald Test (p-value)</b>	<b>0.000</b>		<b>0.061</b>

Notes: AMEs (columns 1–2) are average marginal effects representing the percentage point change in probability. Robust standard errors, clustered at the firm level, are reported in parentheses. The rejection of the null hypothesis in the Wald test ( $p < 0.05$ ) invalidates the proportional odds assumption for the ordered logit model in this context. \*, \*\*, \*\*\* denote significance at the 10 %, 5 %, and 1 % levels, respectively.

An inverse relationship is found between the contemporaneous sectoral cycle—measured by either the output gap or the growth rate—and the probability of a firm being classified in a Ponzi state. This result establishes that a buoyant meso-level economic environment provides a direct and immediate amelioration of corporate financial conditions. The primary causal mechanism operates through the revenue channel: a sectoral expansion translates into superior sales and operational cash inflows for firms, which directly fortifies their capacity to service fixed commitments without recourse to external financing or asset sales. This pro-cyclicality of corporate financial health is a foundational component of the model.

The lagged specifications of the sectoral cycle variables reveal a more intricate temporal dynamic that is central to the Financial Instability Hypothesis. The parameter estimate for the first lag of both the output gap and the growth rate is positive and statistically significant. This finding, while seemingly counterintuitive, is wholly consistent with Minsky's proposition that "stability is destabilizing." The antecedent condition of a prior economic boom fosters euphoric expectations among economic agents, which in turn becomes a proximal cause for an expansion in balance-sheet leveraging and a systematic erosion of solvency margins. This accumulation of liabilities creates a more fragile financial structure that makes the firm more susceptible to financial distress in the current period, even after controlling for contemporaneous economic conditions. This lagged effect captures the endogenous generation of financial fragility born from past prosperity. The second lag of the cycle variables, however, does not present a statistically robust effect. The behavioral consequences of past economic performance on current financial structures have a finite temporal horizon.

The econometric estimations presented in Table 9, which deploy a series of fixed-effects linear probability models, are designed to dissect the relationship between meso-level cyclical dynamics and the incidence of firm-level financial distress. The table is partitioned into two parallel analytical sets: the first block of specifications, models (1) through (4), operationalizes the sectoral business cycle via the Hodrick-Prescott filtered output gap, a measure of an economy's deviation from its secular trend. The second block, models (5) through (8), substitutes this with the sectoral growth rate, a more direct and less processed metric of economic momentum, to function as a robustness test for the primary findings. Across all specifications, a consistent and theoretically coherent narrative of counter-cyclical financial fragility prevails.

The most parsimonious specifications, models (1) and (5), examine the direct contemporaneous correspondence between the state of the sectoral economy and the probability of a firm entering a Ponzi regime. In both models, the parameter estimate for the cycle variable is negative. The causal mechanism is unambiguous: a sectoral expansion, whether captured as a positive output gap or a superior growth rate, translates into an expansion of aggregate demand. This bolsters corporate revenues and operating cash inflows. For a corporate entity with fixed pecuniary commitments, particularly interest payments, this amelioration of the operational cash flow position directly improves its debt-servicing capacity and diminishes the likelihood of a transition into a Ponzi state, where such flows are insufficient to meet interest obligations.

Models (2) and (6) introduce a control for firm scale, operationalized as the natural logarithm of total assets. The parameter on this variable is negative and statistically significant,

which is congruent with established corporate finance theory. Larger corporate entities typically benefit from operational diversification, economies of scale, and preferential access to capital markets, which collectively constitute structural balance-sheet buffers that insulate them from idiosyncratic and systemic shocks. The introduction of this control does not vitiate the economic importance of the sectoral cycle variable because the meso-level economic environment has a direct effect on fragility that is independent of firm scale.

The specifications in columns (3) and (7) incorporate dynamic effects through the inclusion of lagged cycle variables and introduce a control for operational efficiency via capacity utilization. The parameter on capacity utilization is negative and statistically significant. This variable connects a micro-level transmission channel and the meso-level cycle; a sectoral expansion creates the demand conditions that enables a firm to operate closer to its productive frontier. A superior utilization rate spreads fixed costs over a larger output base, which mechanically widens profit margins and expands the internally generated funds available for debt service. The first lag of the cycle variables, Sectoral Output Gap ( $t-1$ ) and Sectoral Growth ( $t-1$ ), returns a positive and statistically significant coefficient. This outcome does not permit us to infer that past expansions cause current fragility. A model that controls for the contemporaneous state of the economy, the lagged positive parameter may capture a mean-reversion tendency in cyclical data or, from a Minskyan perspective, that a sustained period of past expansionary conditions induced an accumulation of balance-sheet leverage that predisposes firms to fragility in the current period. The second lag of these variables is not statistically significant, which denotes that the informational content of economic conditions further in the past is too distal to have a systematic effect on a firm's current solvency status, with its influence already impounded in the firm's extant balance sheet structure.

Finally, the most comprehensive specifications, models (4) and (8), synthesize all previously introduced variables. The central findings remain unaltered. The contemporaneous cycle variable remains negative and statistically significant due to the counter-cyclical nature of proliferation of Ponzi schemes. The controls for firm scale and operational efficiency retain their expected signs. The persistence of these results across the full set of specifications and with two distinct measures of the economic cycle corroborates a robust empirical conclusion: sectoral economic expansions provide a material and immediate improvement in corporate financial status, while downturns are a proximal cause of transitions into financial duress. The dynamics of the lagged variables, however, show that the pathway is not purely mechanical, with the balance-sheet consequences of past cyclical episodes contributing to a firm's antecedent vulnerability.



**Table 8**

Fixed-Effects Linear Probability Model of Ponzi Incidence

Dependent Variable: Ponzi ( =1 )	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sectoral Output Gap ( $Outputgap_{s,t}$ )	-0.894** (0.354)	-0.871** (0.351)	-0.849** (0.388)	-0.828* (0.437)				
Sectoral Output Gap ( $Outputgap_{s,t-1}$ )			0.418* (0.219)	0.409* (0.223)				
Sectoral Output Gap ( $Outputgap_{s,t-2}$ )			0.362* (0.198)	0.351 (0.204)				
Sectoral Growth ( $\% \Delta GVA_{s,t}$ )					-0.621*** (0.189)	-0.597*** (0.186)	-0.582** (0.233)	-0.569** (0.238)
Sectoral Growth ( $\% \Delta GVA_{s,t-1}$ )							0.211** (0.095)	0.203** (0.098)
Sectoral Growth ( $\% \Delta GVA_{s,t-2}$ )							0.092 (0.077)	0.081 (0.079)
Log Total Assets ( $\ln(TA)$ )		-0.024*** (0.008)		-0.025*** (0.008)		-0.023*** (0.008)		-0.024*** (0.009)
Capacity Utilization (CAPUTIL)			-0.112** (0.049)	-0.118** (0.051)			-0.108** (0.048)	-0.115** (0.052)
Firm Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y
Year Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y
Firms	3,376	3,376	3,102	3,102	3,376	3,376	3,102	3,102
Unconditional prob	20.7	20.7	20.7	20.7	20.7	20.7	20.7	20.7

Notes: The dependent variable is a binary indicator for a firm being classified as Ponzi. 'Uncond prob' is the unconditional probability of being Ponzi within the sample. All models include firm and year fixed effects. Robust standard errors, clustered at the firm level, are reported in parentheses. \*, \*\*, \*\*\* denote statistical significance at the 10 %, 5 %, and 1 % levels, respectively.

The unconditional quantile regression estimations detailed in [Table 9](#) afford a granular deconstruction of the transmission mechanism from meso-level economic conditions to firm-level financial stability. The analysis is bifurcated into two panels, each designed to test the central hypothesis under a different specification of the cyclical impulse. Panel A operationalizes the economic cycle via the Sectoral Output Gap, a statistically derived measure of an industry's deviation from its secular output trend. Panel B functions as a robustness check and substitutes this filtered variable with the more direct, un-processed measure of the Sectoral Growth Rate.

The model specifications in Panel A are designed to dissect this heterogeneity. The initial column presents the conditional mean effect via a standard Ordinary Least Squares regression with firm-fixed effects, a baseline of a positive relationship between sectoral expansions and the average firm's Interest Coverage Ratio. The subsequent columns employ the Recentered Influence Function methodology to appraise this correspondence at specific unconditional quantiles of the solvency distribution. The economic rationale for this procedure is to test whether the impact of a common sectoral shock is uniform, or whether it is amplified or dampened by a firm's pre-existing financial structure. The estimations confirm a distinct vulnerability gradient. For firms located in the lower deciles of the Interest Coverage Ratio distribution, the Sectoral Output Gap is a first-order determinant of their financial viability. A positive coefficient of high statistical significance is found, a result consistent with the theoretical premise that these financially constrained entities possess minimal to non-existent operational cash-flow margins and liquidity buffers. Their capacity to satisfy fixed pecuniary commitments is therefore dependent on contemporaneous revenue streams. A sectoral expansion, materializing as a positive output gap, directly ameliorates their binding solvency constraint.

As the analysis progresses across the solvency distribution, this sensitivity is systematically attenuated. For the median firm, the relationship between the sectoral cycle and the Interest Coverage Ratio, while still positive, is of a smaller magnitude and is significant only at a lower level of statistical significance. The typical corporate entity in the sample operates with more financial slack and operational buffers. A sectoral expansion is certainly beneficial, but the marginal impact on its already adequate debt-servicing capacity is modest. The pro-cyclical nature of its financial condition persists, but its elasticity with respect to the sectoral output gap is considerably lower. For firms situated in the upper quantiles of the distribution, the sectoral cycle ceases to be a statistically relevant factor. This outcome is theoretically coherent; these financially unconstrained corporations are profitable, have low leverage, and accumulate internal capital. Their solvency is effectively inelastic with respect to business-cycle frequency oscillations. Their extensive financial buffers absorb the impact of sectoral downturns without any material degradation of their debt-servicing capacity.

The estimations presented in Panel B, which substitute the Sectoral Growth Rate for the Output Gap, serve to corroborate these findings. They are not an artifact of the statistical filtering procedure used to derive the primary cyclical variable. The entire pattern of results is replicated with remarkable fidelity. A strong, positive, and statistically significant association is apparent for the most fragile groups in the lower quantiles. This relationship vanishes entirely for median firms and the most solvent firms in the upper quantiles. The consistency across both panels provides

robust evidence against a uniform conception of pro-cyclical fragility. Instead, the analysis finds that meso-level economic impulses are asymmetrically transmitted through the corporate sector, with their impact amplified by financial fragility and absorbed by financial strength, a mechanism that concentrates the systemic consequences of economic downturns among the most vulnerable constituents.

**Table 9**

RIF Regressions - Effects of Sectoral Cycle on the Interest Coverage Ratio (ICR)

			(1) Mean (OLS-FE)	(2) 10th Percentile	(3) 25th Percentile	(4) 50th Percentile (Median)	(5) 75th Percentile
<b>Panel A: Sectoral Output Gap</b>							
Sectoral	Output	Gap	1.258***	3.014***	1.987***	0.902*	0.419
<i>(Outputgap<sub>s,t</sub>)</i>			(0.593)	(0.988)	(0.712)	(0.513)	(0.604)
<b>Panel B: Sectoral Growth Rate</b>							
Sectoral Growth (% $\Delta GVA_{s,t}$ )			0.882**	2.159***	1.441***	0.653	0.297
			(0.391)	(0.652)	(0.489)	(0.659)	(0.422)
<b>Controls in all models</b>							
Firm Fixed Effects			Y	Y	Y	Y	Y
Year Fixed Effects			Y	Y	Y	Y	Y

*Notes:* The dependent variable is the firm's Interest Coverage Ratio. RIF regressions estimate the effect of the cycle variable on the unconditional quantiles of the ICR distribution. Robust standard errors, clustered at the firm level, are reported in parentheses. \*, \*\*, \*\*\* denote statistical significance at the 10%, 5%, and 1% levels, respectively.

The unconditional quantile regression estimations visually rendered in Panel (a) of [Figure 2](#) deconstruct the effect of the cyclical component of sectoral output on the Interest Coverage Ratio (ICR). The resulting plot uncovers a non-monotonic correspondence between the meso-level economic cycle and corporate solvency. For corporate entities situated in the lower quantiles, a pro-cyclical relationship is apparent. The parameter estimate is positive, of a considerable magnitude, and statistically discernible from zero. The causal mechanism for this segment is the direct revenue channel: a positive deviation of sectoral output from its trend translates into superior operating cash inflows, which mechanically ameliorates the binding solvency constraint for these financially precarious firms. Their ICR is acutely sensitive to contemporaneous revenue streams. This effect attenuates as the analysis moves toward the median of the distribution. For firms situated in the upper quantiles of the distribution, the sectoral cycle ceases to be a statistically relevant factor. The parameter estimate becomes statistically indistinguishable from zero, with wide confidence intervals that comfortably contain the null. This outcome is theoretically coherent. These financially unconstrained corporations are defined by robust profitability, low leverage, and substantial accumulated internal capital. Their capacity to service debt is not a binding constraint. A positive growth shock, while augmenting their profitability, has a negligible

marginal impact on their already-overwhelming ability to meet financial obligations. Their solvency is effectively inelastic with respect to business-cycle frequency oscillations. Their extensive financial buffers absorb the impact of sectoral downturns without any material degradation of their debt-servicing capacity.

Panel (b) presents the results of a parallel unconditional quantile regression analysis, designed to function as a robustness test for the primary findings. This specification substitutes the statistically filtered Sectoral Output Gap with the more direct, un-processed metric of the Real Sectoral GDP Growth rate. The visual output from this estimation confirms the foundational conclusion of an asymmetric transmission of economic impulses, yet it traces a distinct, monotonically decreasing pattern that allows for a more granular dissection of the underlying causal pathways. The graph visually maps a continuous gradient of vulnerability. The elasticity of a firm's solvency with respect to the sectoral economic cycle is an inverse function of its initial financial strength. This monotonic decline provides a clear empirical narrative of how antecedent financial fragility acts as a conditioning variable that governs the propagation of macroeconomic shocks through the corporate sector.

For the cohort of firms located in the lowest deciles of the Interest Coverage Ratio distribution, situated to the left of the "Ponzi Cutoff" demarcation, the effect of sectoral growth is positive, of a large magnitude, and statistically significant. The antecedent condition for these entities is one of extreme financial constraint; they operate with minimal or negative operating cash flow margins, high concentrations of short-term liabilities, and compressed equity cushions that leave them exposed to any adverse shock. The proximal cause of change is a positive sectoral growth shock. The causal pathway is the direct revenue channel. An expansion in sectoral economic activity translates immediately into superior sales and a commensurate expansion of operational cash inflows, or quasi-rents. For an entity whose capacity to meet fixed pecuniary commitments is already compromised, this incremental cash flow is not a peripheral benefit but the principal determinant of its continued viability. It directly widens the margin between cash inflows and contractual outflows, mechanically elevating the Interest Coverage Ratio. The steep negative slope of the coefficient plot in this region of the distribution quantifies this high elasticity of solvency with respect to growth for these financially vulnerable firms.

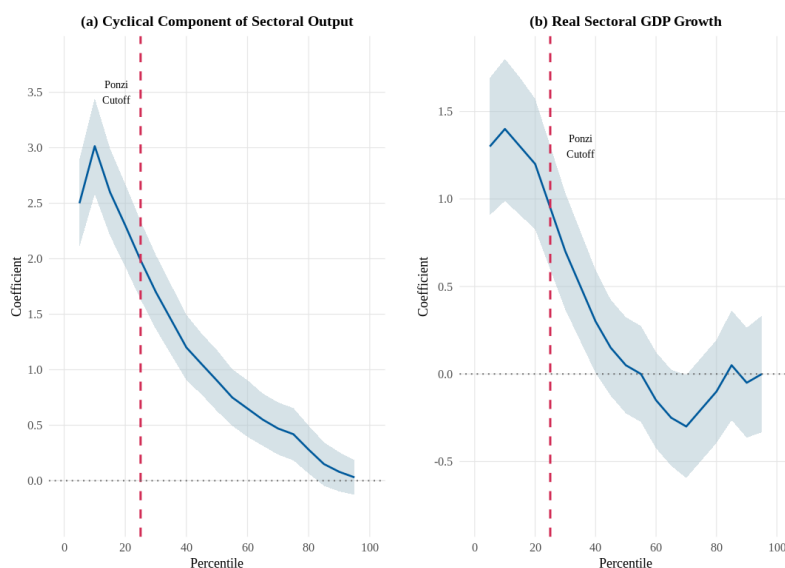
As the analysis moves rightward along the percentile axis, into the median range of the distribution, a structural shift in the transmission mechanism becomes apparent. The slope of the curve flattens considerably, and the parameter estimate, while still positive, contracts in magnitude and its statistical significance attenuates. These firms, typically corresponding to a "Speculative" financing posture, are operationally solvent but require external financing to fund their capital expenditures. A positive growth shock remains beneficial, but its marginal utility for solvency is lower than for the Ponzi group. These firms possess some degree of financial slack. The additional cash flow generated during an expansion is not solely dedicated to averting immediate default; it can be allocated to a wider set of corporate objectives, including discretionary debt reduction, the accumulation of precautionary liquidity buffers, or the internal financing of new investment projects. The effect of growth on the ICR is still positive, but its diminished magnitude and the widening of the confidence intervals show a more heterogeneous set of corporate

responses.

Finally, in the upper quantiles of the distribution, the curve crosses the zero-threshold and the parameter becomes statistically insignificant. The coefficient oscillates around zero, and the wide confidence bands confirm the absence of any systematic relationship. This zone of inelasticity corresponds to financially robust, or "Hedge," corporate entities. These firms display robust cash-flow surpluses, conservative liability structures, and ample liquid asset positions. A positive growth shock, while augmenting their profitability, has a negligible marginal impact on their already-overwhelming ability to meet financial obligations. Their financial health is determined by long-term strategic decisions, capital structure choices, and secular profitability trends, not by short-term cyclical fluctuations. Their balance sheets are structured to absorb such shocks without a discernible impact on solvency metrics. The visual confirmation that their financial stability is effectively orthogonal to the sectoral business cycle is a central finding. This panel therefore rejects a reductive, undifferentiated conception of pro-cyclical solvency and advances a more articulated account in which the positive effects of aggregate expansion accrue disproportionately to the most fragile firms, whereas the financially stable remain largely immune from those variations.

fig2

**Figure 2.** Effects of the output gap by quantile on the ICR



## 4. Discussion

This inquiry assessed the firm-level determinants of financial fragility within Colombian manufacturing enterprises through the analytical prism of Minsky's financial-instability hypothesis. The working proposition maintained that entities dependent upon external liabilities—most notably short-term borrowings and foreign-currency denominated exposures—combined with attenuated operating profitability, would transition into Ponzi-type financing regimes. Empirical estimations corroborated this proposition: observations classified as Ponzi registered markedly

lower profitability ratios and thinner liquidity buffers than Hedge or Speculative counterparts. Ponzi firms recorded operating cash flows insufficient to cover interest expense, which forced repeated debt rollovers. By contrast, Hedge firms produced operational surpluses adequate to satisfy scheduled debt service and planned capital expenditures.

The Castro (2011) taxonomy, which proceeds from a granular, cash-flow-centric calculus, is by its very construction acutely sensitized to the compositional attributes of a corporation's capital account and investment budget. Its definitional thresholds explicitly internalize the nominal exchange rate and its direct impact on debt service capacity. Consequently, econometric covariates that precisely specify a firm's propensity to accumulate liabilities or its structural exposure to foreign currency are expected to operate as primary determinants of its classification within that particular analytical system.

The Nishi (2019) taxonomy is engineered from a margins-of-safety paradigm that functions at a superior level of balance sheet aggregation. Its two definitional pillars are a flow-solvency margin, which computes the ratio of total operating profits to total interest expense, and a stock-based liquidity margin, which computes the ratio of liquid assets to total liabilities. This architecture is intentionally agnostic with respect to the currency composition of those constituent financial accounts. A firm's classification as Speculative or Ponzi within the Nishi system is conditioned by the aggregate relationship between these balance sheet and income statement items, not by the specific currency in which those accounts are denominated. The methodology effectively presumes that all material financial risks, including those that originate from currency mismatches, will ultimately manifest as deteriorations in these aggregate indicators of solvency and liquidity.

When both taxonomies are juxtaposed, an empirical inconsistency is not detected because this divergence originates from the fundamental architectural differences between the two classification methodologies.

Therefore, the statistical insignificance of the external exposure parameters in the Nishi-based model constitutes a theoretically coherent result. It communicates that once a firm's generalized state of flow-solvency and stock-liquidity are empirically controlled for, the marginal information supplied by the currency composition of its liabilities offers no further explanatory power over its fragility classification. This architecture effectively subsumes the risks associated with foreign currency exposure into its broader, top-level metrics of financial health. For instance, a nominal currency depreciation that mechanically inflates a firm's interest expense would be captured through a deterioration of the margin, irrespective of the underlying currency mismatch. The divergence in the computational outcomes thus does not signal a contradiction. The Castro model is specified to explicitly test the direct effects of particular liability structures, while the Nishi model is constructed to appraise a more generalized, and compositionally-agnostic, state of financial robustness.

Given their debt structure, speculative and especially Ponzi entities are sensitive to marginal perturbations in financial conditions. The valuation of these companies is directly related to the capitalization of their expected stream of net profits (minus the interest paid on their liabilities), with an upward trajectory in interest rates that disproportionately impact speculative and Ponzi

schemes as they roll over debt. As institutions of credit enact more rigorous lending criteria, such as a diminution in safety margins, the burden upon speculative and Ponzi companies grows more acute.

Corporate entities burdened by excessive leverage and exposure to the banking system are more susceptible to fluctuations in financial markets. Variations in interest rates or business cycles impinge upon a company's aptitude to administer debt payments, and the increment in leverage ratios compels firms to reorient their focus towards the satisfaction of debt obligations rather than the pursuit of novel investment opportunities (?).

Speculative and Ponzi schemes are beset with heightened volatility in cash flow, more acute problems of asymmetric information, exacerbated agency conflicts between stockholders and lenders, and elevated bankruptcy costs. Hypothetically, these firms ought to preferentially utilize internal funds for their financing requirements. A debt-overhang problem arises when the burden of existing debt on a firm's balance sheet grows so large that the firm faces a high risk of default. Conditional on ex-post financial distress, making a fixed promised debt payment due earlier (i.e., shorter-term) raises the market value of the debt and thus the firm's market leverage and debt overhang ex-post becomes more severe (Rahman, 2023).

Turning to the broader literature, many of these findings align with prior work. The empirical link between compressed profitability, thin liquidity cushions, and corporate fragility echoes Minsky's core insight that operational cash buffers determine solvency margins. Davis et al. (2019) report a secular expansion in the share of Ponzi-classified firms in the United States and emphasize that cohorts unable to service interest from operating income constitute the fragile segment. Iliemena et al. (2019) show that tighter control of receivables and inventory shortens the cash-conversion cycle and strengthens capital-structure resilience among Nigerian manufacturers. Nicolas (2022) documents that short-term funding constraints, operationalized as working-capital pressures, constrain investment by SMEs.

A decrement in profitability metrics corresponds to an augmentation in the probability of a classification as a "Ponzi scheme". In hedge schemes, the improvement of profitability is achieved through optimization of its fundamental drivers: revenues, costs, and capital employed. This dissection allows a circumspect scrutiny of the trade-offs inherent within operational determinations, such as the balance between inventory reduction and equipment efficiency in manufacturing entities (Vernimmen et al., 2017).

Minsky (1991) expounded the well-established tenet of financial instability and asserted that perceived risk does not solely originate from the magnitude of funds, but that, during expansionary phases the perception of solvency alleviates, and there exists a gradual embrace of more hazardous operations. This, paradoxically, renders epochs of placidity and stability destabilizing.

Empirical findings on collateral and foreign-currency liabilities comport with extant credit-market research. Santos and Cincera (2022) identify collateral scarcity and elevated external borrowing costs as binding constraints on firm expansion. The present estimations extend that logic by tracing a statistical association between collateral erosion—proxied by leverage-amplified balance-sheet write downs—and transition into Ponzi status. Ioannidou et al. (2022) show that lenders calibrate exposure to pledged assets; when recoverable collateral values decline, banks



constrict credit provision.

Currency depreciation produced an upward revaluation of FX obligations in local terms, compressed lenders' willingness to extend new facilities, and forced rollovers on vulnerable borrowers; affected firms therefore moved into Ponzi configurations. A negative shock to collateral quality or to FX-denominated debt propagated through the intermediation chain and transmitted solvency pressure to corporate balance sheets.

The empirical findings of the present study resolve into a coherent account of endogenous corporate fragility where incapacity to generate interest-bearing cashflow and a financing structure concentrated in short-dated claims produce state-contingent solvency failures that propagate through balance-sheet channels to system-wide stress. Low interest-coverage ratios function as a proximate indicator of operational inability to service contractual obligations and, when combined with elevated long-run leverage, firms transition from operational self-funding to dependence on continual rollover and external liquidity provision (Brunnermeier and Sannikov, 2014). The causal mechanism operates as follows: an erosion of EBITDA relative to scheduled interest converts discretionary investment into residual claims on future cashflow and tighten the firm's immediate liquidity position and raising the probability of forced asset disposal or debt renegotiation; the ensuing distress causes counterparties to reprioritize credit and cut unsecured lines, which in turn magnifies the initial cashflow shortfall and causes further insolvency pressure (Adrian and Brunnermeier, 2016).

Currency composition of liabilities and dependence on imported capital introduce an orthogonal amplification channel. When a firm's investment program or debt stock is indexed to a foreign currency, an exchange-rate depreciation mechanically raises the local-currency debt service burden and the recorded cost of new capital formation, compressing both the flow-based solvency margin and the stock-based liquidity cushion. The Castro-style open-economy accounting embedded in the taxonomy therefore identifies FX-revaluation as an effective rise in the real interest burden operating through the balance sheet, not merely through trade-competitiveness channels; empirical contrasts reveal that firms with elevated pre-sample imported-capital shares transition to into fragile states during depreciation episodes, consistent with the balance-sheet transmission emphasized in sovereign and corporate dollarization literature (Du and Schreger, 2022). Exchange-rate shocks thus map one-for-one into rollover risk and lower internal funding capacity, converting otherwise transient operating shortfalls into persistent insolvencies when hedging markets are thin or unavailable.

Maturity composition of debt exerts a direct influence on fragility via rollover exposure. Short-term debt shares magnify the speed and severity with which adverse signals (for example, a modest decline in ICR or a negative sectoral output shock) translate into realized funding shortfalls. Empirical estimates place the elasticity of the probability of a Ponzi-state with respect to the share of short-term debt above the elasticity tied to static leverage ratios. This occurs because maturity concentration introduces a nonlinearity. Short-dated liabilities require frequent market validation and thus create repeated rollover events, in which a single refusal to refinance can convert liquidity risk into insolvency risk. This finding aligns with theoretical and empirical analyses that place maturity mismatch at the heart of liquidity-to-solvency cascades

(Medeiros and do Nascimento, 2021; Adrian and Brunnermeier, 2016), adding evidence from a manufacturing-intensive emerging-market sample that institutional limits on long-term corporate financing accentuate this vulnerability.

Internal financing policy—proxied by retention rates and cash holdings—operates as the principal stock-based margin of safety in the dataset. Firms that sustain higher retained earnings and cash reserves exhibit lower transition probabilities into fragile states, holding constant leverage and sectoral conditions. The causal interpretation here is that internal funds reduce reliance on market timing and rollover, compressing the set of adverse realizations that would otherwise force external liquidations. The magnitude of the protective effect is economically meaningful: moving from low to median cash-to-assets ratios lowers the Ponzi probability by several percentage points in average marginal-effect calculations, a margin comparable to the estimated hazard created by a one-unit fall in ICR. This direct trade between internal liquidity buffers and external refinancing dependency echoes Nishi's margins-of-safety perspective and speaks to the microeconomic channel whereby retained earnings operate as shock absorbers, reducing the probability that temporary revenue shortfalls crystallize into solvency events

Sectoral cyclical dynamics generate both contemporaneous and lagged effects that differentiate immediate cashflow responses from medium-horizon balance-sheet accumulation. Positive sectoral output gaps elevate operating cash flows and compress immediate default probabilities; however, one period after sustained expansions the accumulated effect of increased investment financed by external debt raises fragility. This temporally asymmetric pattern concretely operationalizes Minsky's maxim that stability breeds fragility: expansions relax ex-ante financing constraints, induce leverage accumulation and maturity compression as firms opportunistically finance growth, and thereby enlarge the set of contingent liabilities that a downturn can activate. Quantitatively, the contemporaneous output gap coefficient is negative while the first lag coefficient is positive and statistically significant, consistent with models in which procyclical lending and risk-taking generate a delayed increase in default propensity (Brunnermeier and Sannikov, 2014; Forni et al., 2024). Thus, sectoral booms operate as a two-stage causal device: amelioration of short-run solvency followed by a structural thinning of stock-based buffers that raises medium-term fragility.

Heterogeneity analysis clarifies institutional and market structure sources of cross-study divergence. Studies using developed-market samples often report attenuated ICR effects and greater roles for market-based liquidity channels—an outcome attributable to deeper corporate bond markets, derivative hedging capacity, and more robust creditor enforcement that permit refinancing or restructuring without immediate solvency failure. By contrast, this sample displays tighter bank-centric financing relationships and limited access to term capital, which elevate the marginal role of cashflow as a screening metric and amplify the damage that currency shocks and maturity mismatch inflict. The absence of a robust asset-tangibility protective effect in the estimates suggests endogenous collateral devaluation during systemic episodes; collateral cushions fail when asset prices themselves are cyclically depressed or when lenders avoid repossession costs by cutting new credit, converting expected collateral value into illiquidity. This observation resolves an apparent tension with static agency models that render collateral as a stabilizer: in

a networked, cyclical environment collateral values and credit supply co-move such that ex-ante expectations about asset salability dissolve under stress ([Abele et al., 2024](#)).

Network propagation mechanisms provide an integrating explanation for observed systemic amplification. Firm-level fragility clusters within sectors and counterparty networks raise aggregate tail risk, because a local liquidity shortage concentrates counterparty losses and causes margin calls or credit-line withdrawals, which in turn set off second-round effects across suppliers and financial intermediaries. This networked contagion is path-dependent: density of interfirm financial linkages reduces idiosyncratic volatility absorption during small shocks while amplifying propagation when shocks exceed a threshold, a phase transition phenomenon documented in financial-network models ([Acemoglu et al., 2015](#)).

Comparative evaluation with extant literature reveals convergences in mechanism while highlighting methodological reasons for divergence. Papers that analyze cross-sectional bond yields or aggregate corporate spreads capture market price signals that embed forward-looking expectations and sovereign-level feedbacks ([Du and Schreger, 2022](#); [Acharya et al., 2014](#)); panel microeconomic analyses such as the present one isolate within-firm dynamics and therefore uncover flow-to-stock transitions that market prices may partially pre-price. Differences in sample composition, frequency, and identification strategies explain why size or tangibility appear salient in some studies but not here: cross-country panels average over institutional heterogeneity, whereas firm-level within-variation uncovers the causal sequencing from cashflow erosion to leverage accumulation to insolvency.

Overall, the evidence supports a parsimonious yet precise causal thesis: operational cashflow shortfalls, when paired with maturity mismatch and FX exposure in a bank-based financing environment, transmute transient operating shocks into persistent solvency failures; retention of internal liquidity attenuates this conversion; and sectoral booms seed subsequent fragility through balance-sheet accumulation, producing nonlinear network contagion when shocks cross critical thresholds. The micro-to-macro mapping observed in the sample provides empirical substance to Minsky's conceptual architecture and situates corporate fragility as a mechanistic progenitor of broader credit-market instability ([Brunnermeier and Sannikov, 2014](#); [Elliott et al., 2014](#)).

## 5. Conclusion

Financial instability sets in motion a self-reinforcing cyclical mechanism through which various economic factors—including interest rates, asset accumulation, and profit rates—contribute to the aggravation of endogenous crises. Financial constraints amplify this fragility during market contractions. This phenomenon is perpetuated by two interrelated factors: firstly, the pervasive assumption that prevailing business conditions will persist indefinitely; and secondly, a concomitant decline in the demand for capital goods. In the presence of unquantifiable uncertainty, changes in expectations cause endogenous economic cycles, and through this same mechanism, a prolonged crisis can occur. A misalignment between debt structures and cash flow exposes economic entities to liquidity shortfalls and solvency risks.

This research examines how endogenous balance-sheet dynamics, mediated by investment

and financing decisions and amplified by exchange-rate movements, propel Colombian manufacturing firms along a fragility continuum from hedge to speculative to Ponzi positions. Anchored in Minsky's financial instability hypothesis and a cash-flow versus asset-liability perspective, the paper synthesized corporate liquidity dynamics with currency mismatch exposure to fill an empirical gap on open-economy sources of firm-level solvency deterioration.

The paper synthesized two complementary theoretical strands: a cash-flow based conception of financial positions that emphasizes accounting flows and margins of safety, and an asset-liability perspective that highlights currency mismatches and the revaluation of FX-denominated obligations.

The study pursued three explicit objectives. First, it sought to identify the firm-level financial and operational determinants that forecast transitions across fragility states. Second, it aimed to trace the transmission channels, particularly imported capital dependence and foreign-currency liabilities, through which exchange-rate shocks affect solvency. Third, it tested the internal-finance implications of distress, examining whether worsening default risk compresses the internal funds available for investment. The empirical work was conducted on a firm-level panel of Colombian manufacturing firms covering recent business-cycle variation.

Internal financial policy and operational performance are the dominant determinants of a firm's placement on the fragility continuum, since these internal levers dictate the extent to which debt-service capacity withstands adverse cash-flow conditions. A contraction of the interest-coverage ratio is a direct marker of operative incapacity to meet contractual obligations, and this deterioration frequently precedes a shift into a Ponzi scheme through the cash-flow deficit channel, where recurrent reliance on external funding overlays declining operating surpluses. This finding is robust across both the Castro (2011) and Nishi (2019) taxonomies. The comparative analysis of the two models, however, disentangles distinct causal pathways. The Castro specification, designed for an open economy, successfully isolates a specific transmission channel for external shocks; a firm's pre-sample dependence on imported capital registers as a statistically significant predictor of distress, confirming that currency valuation effects on investment costs and foreign-denominated liabilities are a direct conduit of fragility. The Nishi model, in contrast, reveals that when solvency is appraised through a margins-of-safety lens, this variable becomes redundant. The Nishi estimations instead assign paramount importance to the stock-based liquidity buffer, operationalized as the cash-to-assets ratio. This divergence does not represent a contradiction because the Nishi framework subsumes specific balance-sheet exposures into its broader aggregate margins, while the Castro model is specified precisely to isolate them.

The analysis of sectoral cycle dynamics through fixed-effects linear probability models uncovered a temporal asymmetry in the genesis of corporate fragility. The estimations separate the immediate effect of the economic environment from the consequences of prior economic conditions. The results uncover a counter-cyclical pattern in the immediate term. A contemporaneous sectoral expansion, captured by either the output gap or the growth rate, corresponds to a statistically significant lower probability of a firm's classification in a Ponzi state. The causal mechanism operates through the revenue channel; a buoyant meso-level economic environment fortifies a firm's capacity to service fixed commitments. The more preponderant finding, however,

materializes from the specifications that contains lagged parameters. The parameter estimate for the first lag of the cycle variables is positive and statistically significant. This outcome validates the core of Minsky's proposition that "stability is destabilizing." The antecedent condition of a prior economic boom fosters an accumulation of balance-sheet vulnerabilities -likely through an expansion of leverage and a shortening of debt maturities- that elevates the probability of financial distress in the current period, even after controlling for contemporaneous economic conditions. This lagged effect captures the endogenous generation of financial fragility born from past prosperity.

The unconditional quantile regression estimations moved beyond discrete classifications to deconstruct the impact of the sectoral cycle across the entire continuous distribution of interest cover ratio. The salutary effect of a sectoral expansion is not uniform across all firms; its magnitude is an inverse function of a firm's initial financial strength. For the most financially precarious entities located in the lowest quantiles of the Interest Coverage Ratio distribution, a positive sectoral growth shock has a large, positive, and statistically significant impact on their solvency. The economic logic is one of survival. These firms operate on a knife's edge, and the additional cash flow from a cyclical upswing is the primary determinant of their ability to avert immediate default. This sensitivity systematically diminishes for more solvent firms. For corporate entities in the upper quantiles of the distribution, the effect of the sectoral cycle becomes statistically indistinguishable from zero because they forge internal buffers that render their solvency effectively impervious to business-cycle frequency oscillations.

The qualitative synthesis of model estimation exposed a coherent pattern in which progressive reliance on external liabilities to fund capital expenditures and persistent gaps between planned investment and operating cash flow presaged movement away from liquidity-preserving positions toward greater vulnerability. Profitability acts as a stabilizing force, with healthier operating margins associated with persistence in hedge-type positions, while deteriorating margins presage deeper fragility. These qualitative regularities emphasize the centrality of cash-flow capacity in mediating exposure to solvency deterioration.

The study is candid about its limitations. Lack of transparency and inconsistent standardization in the financial statements and accounting documents of particular firms impede a larger sample selection for comprehensive analysis. Furthermore, the data collection period coincided with a systemic crisis, introducing distortions into the financial situation of the selected firms due to the broader economic downturn. These limitations, including the constrained sample size negatively impact the study's findings' generalizability. In addition, the modified distance-to-default measure, while informative, depends on proxies for market valuation and volatility that cannot fully replicate market-based constructs for privately held firms, which may constrain the precision of the inferences about default risk.

By empirically linking weak internal cash flows, aggressive debt financing, and foreign-exchange exposure to the probability of entering a Ponzi regime, it is demonstrated that financial instability is not an exogenous random shock but emerges from firms' own decisions and market structures. This insight advances the field by confirming a core Minskyan insight with micro-data.

Building on these results and limitations, several concrete avenues for future research arise.

Future research endeavors should prioritize areas of inquiry such as the identification of micro-economic determinants associated with financial volatility and elucidation of factors contributing to financial constraints in developing economies, with a specific focus on non-public companies. Unanswered questions include whether these patterns hold outside manufacturing or in countries with different institutional structures. A longitudinal extension that follows firms across a longer and more varied business-cycle sequence would clarify the persistence and reversibility of fragility states; causal identification strategies, including instrumental variables or the exploitation of exogenous policy or trade shocks, could better isolate the direction of causation between financing gaps and fragility transitions; cross-country replications could test the generality of the dual-channel mechanism in alternative institutional settings; and structural modeling that endogenizes investment choice and currency exposure. By pursuing these paths, future research can refine the measurement of default risk in private firms, strengthen causal claims, and evaluate policy levers, such as hedging market development or targeted credit facilities, that might mitigate endogenous fragility.

## Appendix

### Appendix A

This study specifies an econometric model to quantify the influence of a vector of firm-specific and macroeconomic variables on the probability that a given firm occupies one of the discrete, non-ordered financial fragility states: Hedge, Speculative, or Ponzi. A multinomial logit specification is selected for its capacity to handle a categorical dependent variable with more than two outcomes. The adoption of a panel data structure permits the control of unobserved, time-invariant firm heterogeneity.

Let the dependent variable,  $Y_{it}$ , denote the observed fragility state for firm  $i$  at time  $t$ . This variable can assume one of  $J = 3$  values from the set:

$j = 1$  (Hedge),  $2$  (Speculative),  $3$  (Ponzi)

The multinomial logit model selects a base category for reference, against which the probabilities of the other states are estimated. The most stable state, Hedge ( $j=1$ ), operates as the logical baseline. The model then estimates a distinct set of coefficients for each alternative state relative to this baseline.

$$\ln \left( \frac{P(Y_{it} = 2)}{P(Y_{it} = 1)} \right) = \beta'_2 X_{it} + \alpha_{i2}$$

$$\ln \left( \frac{P(Y_{it} = 3)}{P(Y_{it} = 1)} \right) = \beta'_3 X_{it} + \alpha_{i3}$$

The unconditional probability for each state  $j$  is then computed as follows:

$$P(Y_{it} = j) = \frac{\exp(\beta'_j X_{it} + \alpha_{ij})}{\sum_{k=1}^3 \exp(\beta'_k X_{it} + \alpha_{ik})}$$



For the base category (Hedge,  $j=1$ ), the coefficients are normalized to zero:  $\beta_1 = 0$  and  $\alpha_{i1} = 0$ .

The Financial Fragility State is a categorical variable classifies firm  $i$  at time  $t$ . The model will be estimated twice, each time employing a different operational definition for this conceptual variable to test the robustness of the findings.

The three Minskyan states are discrete, non-ordinal values assumed by  $Y_{it}$ .

$j = 1$  (Hedge): The most robust financial position; designated as the base category for coefficient interpretation.

$j = 2$  (Speculative): An intermediate position where operating cash flows cover interest obligations but not principal repayments on investment outlays.

$j = 3$  (Ponzi): The most precarious financial position, where operating cash flows are insufficient to meet interest commitments.

The Vector of Independent Variables comprises  $k$  explanatory variables for firm  $i$  at time  $t$ . Its composition remains constant across all model estimations.

? extends Minsky's model to encompass an open economy framework. In this theoretical construct, the transition between financial positions is endogenized by permitting the investment rate to surpass the profit rate. A firm is deemed insolvent when its net worth is less than or equal to zero. The financial fragility of the system emerges endogenously, influenced by the trajectories of three variables: interest rates ( $i$ ), profit ( $r$ ), and growth ( $g$ ).

The erosion of creditor confidence is caused by the reallocation of resources from productive to speculative investments. This shift is motivated by pervasive uncertainty or the anticipation of inferior returns from productive investments relative to their speculative counterparts. The deterioration of confidence is exemplified by the reluctance of creditors to extend credit facilities.

The Financial Instability Hypothesis (FIH) examines the fragility inherent in the financial system through a macroeconomic lens. Cash flow can be disaggregated into its constituent components which comprises funding sources, net operating income,  $R$ , and new loans,  $D$  which are subsequently allocated to finance investment activities,  $I$ , and debt servicing obligations,  $V$ :

$$R + D = I + V$$

The aforementioned accounting identity embodies a degree of arbitrariness because the variables involved can assume negative values under certain conditions. For instance, ( $R$ ) can become negative if the company experiences operational losses. Similarly, debt ( $D$ ) can attain negative values when an entity actively reduces its outstanding liabilities. Moreover, investments ( $I$ ) may register as negative when an entity engages in asset divestiture, and ( $V$ ) can turn negative if the entity transitions to a net creditor position.

The net worth of a company, denoted by  $W$ , is calculated by subtracting the value of its debts,  $B$ , from the value of its assets,  $A$ . The growth of equity, represented by  $\dot{W}$ , is equivalent to the difference between the growth rate of its assets,  $\dot{A}$ , and the growth rate of its debts,  $\dot{B}$ :

$$W = A - B$$



$$\dot{W} = \dot{A} - \dot{B} = I - D$$

The growth rate of the company's assets ( $g$ ) can be expressed as the ratio of investments (I) to assets (A),  $g = I/A$ . The profit rate ( $r$ ) is equivalent to the ratio of revenue (R) to assets (A). The interest rate ( $i$ ) is defined as the relationship between debt service and the volume of debt,  $i = V/B$ .

In the event that a bankrupt company is deemed insolvent,  $W \leq 0$ , its creditors will be left with no recourse to recover the principal of their loans.

Then the cash flow identity can be rewritten as:

$$\dot{B} = D = I + V - R = (g - r)A + iB$$

Consider trajectories where the growth rate ( $g$ ) remains constant. In such scenarios, assets evolve according to  $A(t) = A_0 e^{gt}$ . The general solution of this differential equation is:

$$B(t) = B_0 - \frac{g-r}{g-i} A_0 e^{it} + \frac{g-r}{g-i} A_0 e^{gt}$$

For analytical convenience, this solution is expressed in terms of the debt/assets ratio,  $\phi = B/A$ .

The trajectory of  $\phi$ , denoted as  $\phi^* = (g-r)/(g-i)$ , is given by:

$$\phi(t) = \phi^* + (\phi_0 - \phi^*) e^{(i-g)t}$$

If  $g > i$ , then the second term disappears asymptotically, and  $\phi(t) = \phi^*$ .

If  $i > g$  then the second term dominates, and  $\phi(t) = \pm\infty$  as  $\phi_0 > < \phi^*$ .

A company becomes insolvent within a finite time frame under two conditions: where  $g > i$  and  $\phi^* > 1$ ; and  $i > g$  and  $\phi_0 > \phi^*$  (a progressive deterioration of the debt/assets ratio).

In trajectories where  $r > i$ , the company avoids bankruptcy.

When  $r > g > i$ ,  $\phi(t) \rightarrow \phi^* < 0$ , the company becomes an asymptotic net creditor, with the debt/assets ratio turning negative over time.

When  $r > i > g$ ,  $\phi^* > 1 > \phi_0$ ,  $\phi(t) \rightarrow -\infty$  and the entity becomes an asymptotic creditor over time under a hedge scheme.

When  $g > r > i$ ,  $\phi(t) \rightarrow \phi^* < 1$ , the company avoids a precarious debt position.

The satisfaction of payment obligations through the sale of assets or the modification of financial liabilities receives the designation "position formation." This process entails the possession of assets that, despite generating income, are not readily convertible to cash without frictions within financial markets. Due to the restricted courses of action during periods of income contraction, firms endeavor to vary the composition of their asset portfolios via debt renegotiation or the sale of assets.

The operationalization of the three financial states depends on a meticulous comparison of a firm's principal source of internal capital (operating cash inflows,  $R$ ) against its primary applications of capital (gross capital formation,  $I$ , and debt service,  $V$ ). The ? model endogenizes currency risk by defining  $V$  as  $i(B_d + eB^*)$ .

$$R \geq I + i(B_d + eB^*)$$

A Hedge unit is characterized by a condition of complete financial self-sufficiency relative to its current operational and investment cycle. The logic embedded within this inequality is one of a positive net cash flow subsequent to all commitments.

The corporation's operating cash inflows ( $R$ ) are of sufficient magnitude to sequentially satisfy two discrete strata of cash outflows. Initially, they cover the non-discretionary, contractual debt service payments ( $V$ ). The residual cash flow then remains adequate to cover the discretionary, yet strategically vital, expenditures for gross capital formation ( $I$ ).

A firm in a Hedge position generates an internal financing surplus. This surplus,  $S_I = R - I - V \geq 0$ , confers upon the firm maximum financial autonomy. It can apply this surplus to:

De-lever: Actively amortize the principal of its extant debt stock, thereby contracting future financial obligations.

Accumulate liquid assets: Augment its treasury reserves or marketable securities, thus fortifying its balance sheet's resilience against unforeseen shocks.

Distribute profits: Remit larger dividend payments to shareholders without compromising its financial stability.

This posture is typically associated with mature corporations in stable industries, or with firms at the inception of a new investment cycle where initial projects are exceptionally profitable and leverage is minimal. A Hedge unit's viability is not contingent upon the state of capital markets; it can execute its business plan even during a credit contraction.

$$i(B_d + eB^*) \leq R < I + i(B_d + eB^*)$$

A Speculative unit operates in a state of conditional solvency. Its financial structure is viable only under the presumption that external capital markets will remain accessible and accommodating. The logic is one of a positive operating cash margin but a negative investment cash flow.

The company's operating cash inflows ( $R$ ) are adequate to satisfy the primary stratum of its commitments—the contractual debt service payments ( $V$ ). However, after servicing its debt, the remaining cash flow is inadequate to fund its planned capital expenditures ( $I$ ). This circumstance creates a structural financing gap,  $G_F = I - (R - V) > 0$ .

To bridge this financing gap, the corporation must consistently access external capital markets. Its principal strategies involve: rolling over debt, i.e. refinancing maturing principal obligations with new debt issuance. Or alternatively, issuing new debt by securing additional credit facilities to fund the portion of the capital budget not covered by internal resources.

This posture is the very engine of a Minskian expansion. It characterizes firms in a growth phase, where optimistic expectations about future profits ( $p$ ) justify levels of investment ( $g$ ) that outpace current cash generation ( $r$ ). The firm is solvent on an operating basis, but its growth trajectory is entirely dependent on the continuous validation of its optimism by its creditors. Its paramount vulnerability is refinancing risk and interest rate risk. An abrupt escalation in interest rates or a constriction of credit availability can render its position untenable.

$$R < i(B_d + eB^*)$$

A Ponzi unit is defined by a condition of fundamental insolvency. Its financial structure is inherently unstable, as its core operations fail to generate sufficient cash to satisfy even the most immediate financial obligations. The underlying logic is one of a negative operating cash margin.

The enterprise's operating cash inflows ( $R$ ) are so deficient that they are unable to cover even the first and most critical stratum of cash outflows—the interest and principal payments on its debt ( $V$ ). The enterprise is, in effect, “cash-flow negative” prior to any consideration of new investment. To remain a going concern, a Ponzi unit must finance not only its capital investments but also a portion of its interest payments. This is accomplished by:

Capitalizing interest: issuing new debt solely to make interest payments on pre-existing debt, which results in an exponential growth of the total debt stock,  $B$ .

Selling assets: liquidating productive assets to generate cash for debt service, which erodes its future cash-generating capacity.

Relying on asset price appreciation: the firm's viability is predicated on the expectation that the market value of its assets will appreciate faster than its debt accumulates, allowing an eventual sale of assets at a profit to clear its obligations.

This represents the terminal stage of financial fragility. The enterprise is no longer investing for future production but is engaged in a purely financial operation to avert bankruptcy. Its existence is a wager on a continuously accommodative financial environment or a speculative asset bubble. Any tightening of monetary policy or downturn in asset prices will ordinarily precipitate its collapse.

### III. The Dynamic Trajectories to Insolvency

The dynamic analysis of the debt-to-asset ratio,  $\phi(t)$ , supplies the temporal dimension to the static classifications. The evolution of this ratio identifies the precise pathways to corporate failure. The governing equation is:

$$\phi(t) = (\phi_0 - \phi^*)e^{(i+\epsilon\delta-g)t} + \phi^*$$

A firm's long-term solvency is determined by the interaction between two factors: the sign of the exponent in the exponential term, which establishes whether the system is convergent or divergent, and the value of the steady-state debt ratio,  $\phi^*$ , which informs on the equilibrium point of that convergence.

### IV. The Two Finite-Time Bankruptcy Trajectories

Castro's work identifies two discrete scenarios that culminate in insolvency:

Divergent Insolvency:  $g < i + \epsilon\delta$  and  $\phi_0 > \phi^*$

The condition  $g < i + \epsilon\delta$  defines a circumstance where the rate of asset growth ( $g$ ) is inferior to the effective cost of debt financing. The effective cost is the summation of the nominal interest rate ( $i$ ) and the capital loss from currency depreciation ( $\epsilon\delta$ ). In this case, the firm's liabilities expand at a faster rate than its asset base. This renders the exponent in the dynamic equation positive, causing the system to be unstable and divergent.

The path  $\phi_0 > \phi^*$  shows that if the firm's initial debt ratio ( $\phi_0$ ) is already less favorable (higher) than its unstable steady-state equilibrium ( $\phi^*$ ), no force exists to restore it to a stable trajectory. The positive exponent functions as an accelerator. The debt-to-asset ratio,  $\phi(t)$ , will grow exponentially over time, inevitably transgressing the bankruptcy threshold of  $\phi = 1$ . This trajectory depicts a firm already in a precarious position whose financing costs outpace its growth, culminating in a rapid and explosive deterioration of its balance sheet.

Convergent Insolvency:  $g > i + \epsilon\delta$  and  $\phi^* > 1$

The condition  $g > i + \epsilon\delta$  defines a circumstance where the rate of asset growth is superior to the effective cost of debt. This renders the exponent in the dynamic equation negative, meaning the system is stable and convergent. The firm's debt ratio will perpetually tend toward its steady-state value,  $\phi^*$ .

In the path  $\phi^* > 1$  lies the paradox. Although the system is dynamically stable, the equilibrium point itself is one of insolvency. The steady-state debt ratio,  $\phi^* = (g - r)/(g - i - \epsilon\delta)$ , exceeds 1. This occurs when the firm's profit margin is chronically depressed (a small  $g - r$  spread) relative to its growth-adjusted financing cost (a small  $g - i - \epsilon\delta$  spread). The firm is on a stable path, but it is a path that converges directly to bankruptcy. This represents a "zombie" firm scenario, where the company can continue to operate and invest for a time, but its underlying business model is fundamentally unprofitable and its long-term fate is sealed.

The model also provides a depiction of the behavior of financially viable, yet fragile, firms.

The condition for long-term solvency is  $r > i + \epsilon\delta$ . This inequality ensures that the rate of profit ( $r$ ) exceeds the effective cost of debt. This guarantees that the steady-state debt ratio,  $\phi^*$ , will be inferior to the rate of investment ( $g$ ), which is a condition for long-term viability. A firm that satisfies this condition will not become bankrupt if it maintains its parameters.

The speculative corridor  $g > r > i + \epsilon\delta$  is the precise definition of a sustainable, yet speculative, financial posture.

$r > i + \epsilon\delta$ : The firm is profitable enough to be solvent in the long run.

$g > r$ : The firm is investing at a rate that outstrips its internal profit generation. This firm is actively pursuing a high-growth strategy financed by debt. It is in a *Speculative* position by definition. Its wager is that future profits from its aggressive investment will eventually elevate its rate of profit,  $r$ , permitting it to transition back to a more stable *Hedge* position. It constitutes a calculated risk.

The precipitant for a Ponzi transition  $g > i + \epsilon\delta > r$  is the condition that instigates the descent into a Ponzi state. The firm continues to grow its asset base faster than its financing costs ( $g > i + \epsilon\delta$ ), which might project an illusion of health. However, its rate of profit,  $r$ , has now fallen below the effective cost of its debt. This event can be set in motion by three discrete shocks:

- **An elevation in the interest rate ( $i$ ):** A monetary tightening by the central bank.
- **An acceleration in the rate of devaluation ( $\epsilon$ ):** A sudden, adverse movement in the exchange rate.
- **A contraction in the rate of profit ( $r$ ):** A recession or a deterioration in the firm's competitive standing.

When this condition is met, the firm's cash inflows from operations ( $R$ ) are no longer adequate to cover its debt service payments ( $V$ ), even though it might still be investing. It has transitioned from a sustainable speculative wager to an unsustainable Ponzi scheme.

## Appendix B

Nishi's alternative specification to ascertain the robustness of the primary conclusions, this investigation deploys a competing classification methodology for financial fragility states. This alternative operationalization of the dependent variable follows the margins-of-safety paradigm developed by ?, constructing a distinct categorical variable from a different theoretical standpoint. The analysis then subjects this second classification scheme to the identical multinomial logit panel estimation procedure previously detailed. This methodological triangulation permits a direct comparison of the coefficient vectors and their statistical properties across both specifications, offering a rigorous check on the stability of the parameter estimates and validating the structural influence of the selected determinants.

? characterized financial fragility as resting upon three distinct margins of safety: the cash flow margin  $\tau$ , the capital value margin  $\mu$ , and the liquid asset kicker  $\eta$ . The analysis herein focuses on the capital value margin, which derives from the interrelation of contractual debt payments and operational receipts.

Cash flow margin  $\tau$  is closely related to capital value margin  $\mu$ . The formalization of capital value margin is established from the relationship between contractual cash payment commitments ( $CC$ ) and quasi-rents ( $Q^e$ ) and its variance  $\sigma_Q^2$ . The present value calculation applies discount operator  $K$  to these variables. While Minsky's original formulation included time subscript  $i$ , the current analysis abstracts from temporal considerations for analytical clarity. The scalar term  $\lambda$  functions as a multiplier on quasi-rent fluctuations, though Minsky's original work did not fully elucidate its theoretical role.

The mathematical expression for the margin of safety in capital values takes the form:

$$P_k = \mu K(CC),$$

wherein  $P_k$  represents the capitalized value of expected quasi-rents, calculated as:

$$P_k = K(Q^e - \lambda \sigma_Q^2).$$

This ratio places the capitalized value of contractual cash payment commitments against the capitalized value of expected quasi-rents.

The complete expression for the margin of safety in capital values thus becomes:

$$\mu = \frac{K(Q^e - \lambda \sigma_Q^2)}{K(CC)}$$

The magnitude of  $\mu$  directly corresponds to the robustness of the margin of safety – a higher value of  $\mu$  denotes a more substantial safety margin.

For hedge units, the margin consistently exceeds unity across all periods. For a speculative unit this relationship depends upon both the discount rate and the temporal dimension. The

mathematical condition  $P_k > K(CC)$  remains satisfied, yet its fulfillment becomes contingent upon these parameters.

In this calculation, the margin of safety in capital values is approximately obtained as the current value term:

$$\mu = \frac{Q^e - \sigma_Q^2}{CC},$$

for the reasons mentioned above.  $\sigma_Q^2$  is calculated as the variance in operating profits during the time window. The margin of safety in the liquid asset kicker is obtained by

$$\eta = \frac{K(CC) + B_d - P_k K}{k(CC)}.$$

Minsky's employment of  $K(CC)$  seems to denote not the capitalized value of contractual cash payment commitments on debts but rather liabilities on the balance sheet.

The margin of safety in capital values falls below unity in Ponzi schemes, i.e., the capitalized value of contractual cash payment commitments exceeds the capitalized value of expected quasi-rents, a condition that persists without temporal exceptions.

Minsky also mentioned the importance of balance sheet conditions to cover accidental deterioration in cash flow. The Nishi's financial fragility index in terms of the balance sheet is constructed through this lens. It introduces the concept of precautionary liquidity holdings, quantified as  $\eta K(CC)$  or of other liquid assets in addition to the  $P_k K$  of capital assets, where liquid assets act as a buffer against operational shortfalls. This connects with Keynesian theory regarding the prudence of accumulating assets denominated in the same form as obligations.

A unit will own money and marketable financial assets beyond what is needed for transactions as an implicit insurance mechanism against operational cash flow deterioration. Thus, a balance sheet of a hedge finance investor will include  $\eta K(CC)$  of money or of other liquid assets in addition to the  $P_k K$  of capital assets; this money or liquid assets are not needed by the operation of the unit. The balance sheet of a hedge unit can be characterized by

$$P_k K + \eta K(CC) = K(CC) + Eq, \quad \eta \geq 1, \text{ or } \eta \leq 1$$

where  $Eq$  is the equity and  $\eta$ , which will be called a liquid asset kicker, is the measure of the margin of safety in assets superfluous to operations.

As the margin of safety by the liquid asset kicker is obtained by

$$\eta = \frac{K(CC) + Eq - P_k K}{K(CC)},$$

in a simple form, it measures the liquid assets-to-liabilities ratio on the balance sheet. In general, the higher this ratio for an economic unit is, the more robust it is against accidental financial shocks.

In terms of the liquid asset kicker derived on the basis of the balance sheet, the financial fragility of hedge and speculative units is defined as follows. For a hedge unit, liquid asset kicker  $\eta$  is between zero and unity or is greater than unity for all periods. For a speculative unit, the liquid asset kicker is smaller than unity for some periods. With regard to Ponzi finance, Minsky

did not explicitly refer to liquid asset kicker  $\eta$ . As for the balance sheet conditions, he only mentioned that Ponzi finance involves a continuous erosion of equality  $dEq/dt < 0$  over time (Minsky 1986, p. 341). Since the criterion for a Ponzi scheme is unclear in terms of the liquid asset kicker, Nishi's model distinguishes a Ponzi unit by exclusively using the margin of safety in capital values instead of the liquid asset kicker.

Financial fragility is defined by taking both the margin of safety in capital values and the liquid asset kicker into consideration. The definition is summarized as follows:

- **Hedge:** The economic unit is a hedge unit if the margin of safety in capital values is  $\mu > 1$  and the liquid asset kicker is  $\eta > 1$ .
- **Speculative:** The economic unit is a speculative unit if the margin of safety in capital values is  $\mu \geq 1$  but the liquid asset kicker is  $0 < \eta \leq 1$ .
- **Ponzi:** The economic unit is a Ponzi unit if the margin of safety in capital values is  $\mu < 1$  regardless of the value of the liquid asset kicker.

To calculate the margin of safety, two assumptions for simplicity are made. First, financial fragility is defined for each year without considering future periods. For example, Minsky (1986) defined hedge finance as the position that quasi-rents are sufficiently larger than the contractual cash payment commitments on debts for all periods. He also considered different discount rates and the time periods during which the financing units expect cash payments on debt to exceed cash receipts from operations to define speculative and Ponzi positions. Since it is not possible to ascertain these periods and calculate different discount rates on the basis of sectoral statistics, the financial fragility of a sector is captured by focusing on the financial statements each year. Second, he employed the capitalized value (present value) of expected quasi-rents and the cash payment commitments by capitalizing operator  $K$  in his theory of financial fragility taxonomy. By contrast, a finance position is considered on the basis of the definition of current and realized value. In empirical analysis, the capitalized value cannot be calculated because distinguishing what types of capital assets and debt items generate future profit and debt service and the length of the periods is impossible. Moreover, Minsky did not give an exact form of capitalizing operator  $K$ . Thus, these assumptions mean that fragility indicators in this study provide information based on the past and present situations of financial fragility.



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