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DETECTION OF EPILEPTIC EVENTS IN EEG USING WAVELETS

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Abstract:
This paper deal with the problem of automatic detection of epileptic events in EEGs from depth electrodes using multiresolution wavelet analysis. The basic problems in events detection are considered: the time localization and characterization of epileptiform events, and the computational efficiency. The algorithm presented is based on a polinomial spline wavelet transform. The multiresolution representation obtained from this wavelet transform and the digital filters derived allow us an automatic detection, efficient and fast, of epileptiform activity. The detector proposed is based on the multiresolution energy function. This paper shows that it is possible to use a multiresolution wavelet scheme for detecting events in a nonstationary signal. EEG records from depth electrodes were analysed and the results obtained are shown.

Keywords: *wavelets, EEG, digital filters, events detection.*

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1 Introduction

Records of brain electrical activity from depth electrodes are used to localize the origin of seizure discharges in epileptic patients who are candidates for surgical removal of the seizure focus. In clinical practice, the epileptogenic loci are infered from the visual analysis of the interictal and ictal discharges. Automatized systems may be used to detect epochs of the signal that contain transients, patterns and features characteristic of abnormal conditions. There are two basic areas of clinical application: an automatic system may be used as a data reduction process in long term EEG monitoring or as a detector of a satisfactory amount of epileptic transients.

Several techniques have been applied in order to solve the problem of computer assisted detection of epileptiform transients. They include: template matching [7]; parametric [1], mimetic [9] and syntactic [16] methods; neural networks [6]; expert systems [8]; phase-space topography [10] and wavelets [13], [14].

As it was recently pointed out [5], the electroencephalography -despite its widespread use- it is one of the last routine clinical procedures to be fully automated.

This paper intends to provide a contribution to the field by introducing an efficient approach for detecting epileptic events in EEG automatically. It is based on the spline wavelet transform recently introduced by Unser et. al. [15], which was found especially suited for the digital treatment of non stationary signals [3]. As we will show, the analysis based on this wavelet provides a solution to the problems considered basic in events detection:

1. good localization of the information in the time-frequency domain,
2. characterization of the different types of epileptiform events,
3. real-time implementation of the algorithm proposed.

At this point we will do some considerations on the wavelet framework proposed in this paper.

1. The time-frequency localization is optimal, i.e. the area of the window in the time-frequency domain takes the minimum value according to the incertanty principle (Section 2).

2. The computational burden for calculating the wavelets coefficients is minimum, the digital filters derived from the spline wavelet analysis are efficient, and a recursive from can be used [17].

3. The spline wavelet analysis defines two biorthogonal basis and, in consequence, we need to calculate two coefficients per sample in order to get the values of the energy function (Section 3). This calculation is very important in the present scheme since the detector is based on the energy function; we use the fast algorithm introduced by Unser et. al. [15]. We point out that other wavelets can be used, but the main result presented in this paper is that *an algorithm based on multiresolution analysis, biorthogonal or not, can be useful for detecting transients*. The reasons for using polinomial spline wavelets were stated above, and they refer to optimal localization properties and computational efficiency.

4. The multiresolution is dyadic, i.e. the scale parameter is 2^j and the frequency axes is partitioned in octaves (Section 2). As it is well know, in every level of the multiresolution analysis we get a half of the wavelet coefficients with respect to the previous level; for example, in our case, 256 in the first level, 128 in the second, and so on. This fact represents a limitation of the dyadic scheme for detection proposes ([18]). However, as we state above, inside the multiresolution framework the most efficient algorithms were developed. In order to use a fast multiresolution algorithm with good properties for detecting transients, we introduce in Section 3 a simple decomposition of the energy function in each resolution level. In this way we obtain an efficient event-detector based on digital filters designed from a dyadic scheme.

We have used the algorithm presented in this paper for analizing EEG signals from epileptic patients who are candidates for surgical removal of the seizure focus. The EEGs have been recorded from *depth electrodes*, and all the results presented in this paper correspond to this type of signals.

The organization of the paper is as follows. In Section 2 the spline wavelet analysis is introduced and its links to the above mentioned items are shown. (For more details about wavelet theory see References [2], [4], [11], [12]). The notion of energy function and the detection algorithm are considered in Section 3. Finally, results obtained from EEG records of epileptic patients are presented in Section 4.

2 Wavelet analysis

Suppose that $\psi(t) \in L^2(\mathbb{R})$ is any basic wavelet, i.e. it verifies [11]

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

The integral wavelet transform of a finite energy signal $f(t)$ is defined by

$$(W_\psi f)(b,a) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t)\, \overline{\psi\left(\frac{t-b}{a}\right)}\, dt \tag{1}$$

Using as basic wavelet a complex exponential function plus a Hanning window compactly supported, Senhadji et.al. [13], [14] have shown the usefulness of the wavelet analysis in the detection of spikes and spikes-and-waves in EEG. The algorithm proposed by these authors calculates the wavelets coefficients using a discretized version of the integral wavelet transform (1). This implies the calculation of an approximate integral for each pair of parameters a,b chosen, i.e. it is a method computationally expensive. Trying to get a numerically efficient and fast enough events detector, we propose an algorithm based on digital filters.

With this in mind we analyse the event detection problem in a different framework, given by the polynomial spline wavelet transform recently introduced by Unser et. al. [15]. The multiresolution representation obtained from this wavelet transform and the digital filters derived allow us the automatic detection of epileptiform activity in such a way that the requirements enumerated in the Introduction are verified.

The recursive algorithm for calculating the wavelet coefficients is given by the formulas (5) to (9). The decimation used in the algorithm can be seen as a drawback when we are looking for transient events. We have overcame it introducing the detector defined in Section 3, which is adequately defined for compensating the effects introduced by the decimation factor.

Summing up, in order to solve the events detection problem with the two basic constraints imposed in the Introduction we chose the multiresolution representation given by Unser et. al. Since this approach uses a decimation factor that impair the time localization of the events, we introduce a detector based on the energy function (10) considering a uniform distribution of the atoms of the total energy.

The results obtained by processing EEG records from depth electrodes with the proposed algorithm were considered satisfactories (Figures 1 to 5 show typical examples).

We will use as scaling function the cubic spline compactly supported

$$\phi(t) = \begin{cases} 1 - |t| + (1/6)|t|^3 - (1/3)(1-|t|)^3 & \text{si } |t| \le 1 \\ (2-|t|)^3/6 & \text{si } 1 \le |t| \le 2 \\ 0 & \text{si } |t| > 2. \end{cases} \qquad (2)$$

The corresponding wavelet function $\psi(t)$ ([15]) is:

$$\begin{aligned} \psi(t) = \frac{1}{40320}[&-\phi(2t+6) + 124\phi(2t+5) - 1677\phi(2t+4) + \\ &+ 7904\phi(2t+3) - 18482\phi(2t+2) + 24264\phi(2t+1) - \\ &- 18482\phi(2t) + 7904\phi(2t-1) - 1677\phi(2t-2) + \\ &+ 124\phi(2t-3) - \phi(2t-4)]. \end{aligned}$$

In order to analize the time-frequency localization properties of the analysis, we will calculate the values of the center and radius of the time and frequency windows:

$$m_\psi = \frac{1}{\|\psi\|_{L^2}^2} \int_{-\infty}^{\infty} t|\psi(t)|^2\, dt = -0.5$$

$$\Delta_\psi = \frac{1}{\|\psi\|_{L^2}} \left[\int_{-\infty}^{\infty} (t+0.5)^2 |\psi(t)|^2\, dt\right]^{1/2} = 0.5419$$

$$m_{\hat\psi} = \frac{1}{\|\hat\psi\|_{L^2}^2} \int_{-\infty}^{\infty} \omega\, |\hat\psi(\omega)|^2\, d\omega = 5.1632$$

$$\Delta_{\hat\psi} = \frac{1}{\|\hat\psi\|_{L^2}} \left[\int_{-\infty}^{\infty} (\omega - 5.1632)^2 |\hat\psi|^2(\omega)\, d\omega\right]^{1/2} = 0.9239.$$

From these calculations we obtain the value of the window area in the time-frequency plane:

$$(1/4)area = \Delta_{\hat\psi}\Delta_\psi = 0.50067,$$

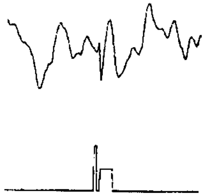

Figura 1

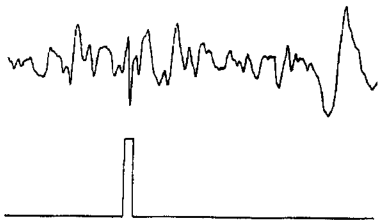

Figura 2

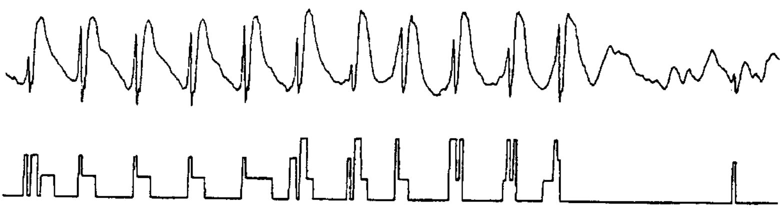

Figura 3

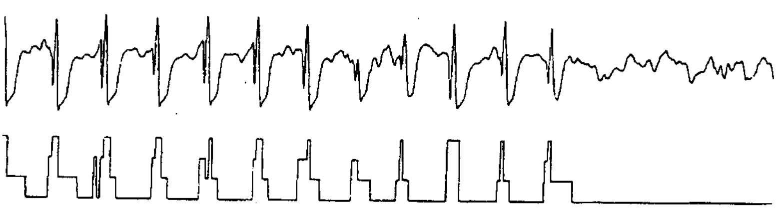

Figura 4

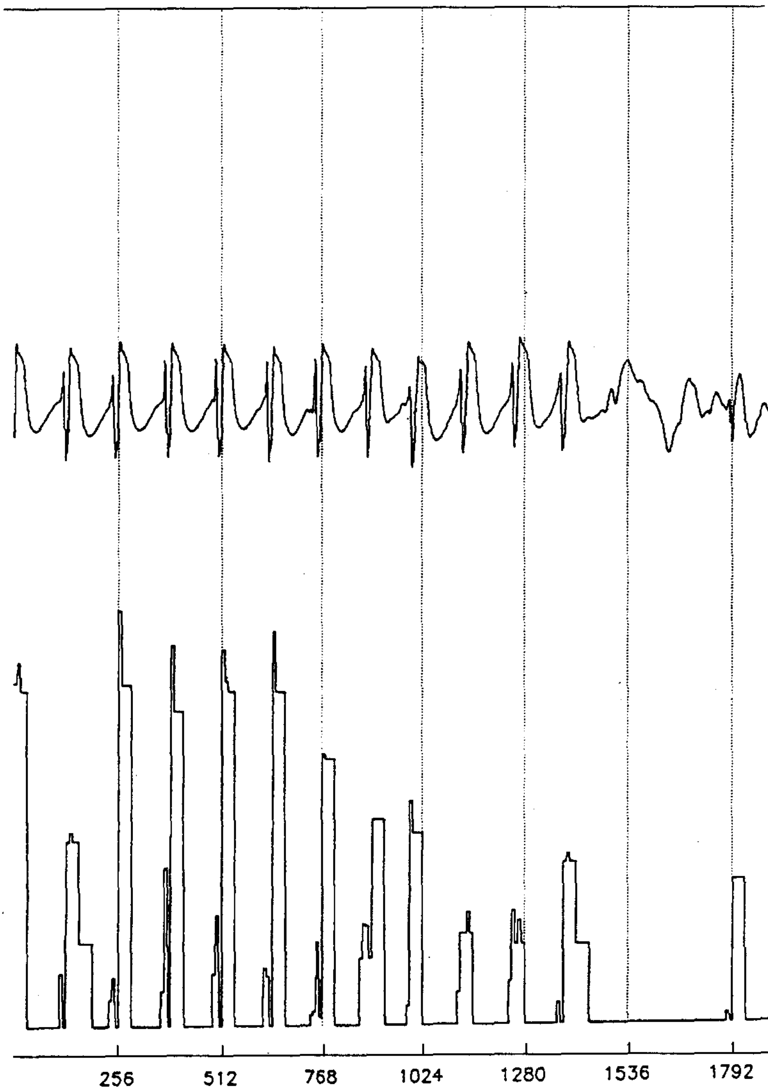

Figura 5

i.e. almost the optimal value 0.5 (Cf. [2]). Thus, the selection of this wavelet guarantee a good localization in the time-frequency plane, which was the first of the key points enumerated in the Introduction. The second point, i.e. the numerical implementation of the algorithm, will be considered in the following.

For a given EEG signal $s(t)$ initially represented by its polynomial spline coefficients at resolution 0, the wavelet decomposition is

$$\begin{aligned} s(t) &= \sum_{k=-\infty}^{\infty} c_0(k)\phi(t-k) = \\ &= \sum_{k=-\infty}^{\infty} c_N(k)\phi(2^{-N}t-k) + \sum_{j=1}^{N}\sum_{k=-\infty}^{\infty} d_j(k)\psi(2^{-j}t-k), \end{aligned} \tag{3}$$

where the numbers $d_1(k), d_2(k), \ldots, d_N(k)$ are the wavelet coefficients, and the sequence $\{c_N(k)\}$ represents the coarser resolution signal at resolution level N. If this decomposition is carried out over all resolutions levels, the wavelet expansion

$$s(t) = \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} d_i(k)\psi(2^{-j}t-k) \tag{4}$$

is obtained. In each level j the series in (4) has the property of complete oscillation [2], which makes the decomposition useful in applications to localization.

At this point it is convenient to introduce two digital filters that will be used in the algorithm. They are given by the transfer functions

$$B^{-1}(z) = \frac{6}{z+z^{-1}+4}$$

$$A^{-1}(z) = \frac{5040}{z^3+z^{-3}+2416+1191(z+z^{-1})+120(z^2+z^{-2})}.$$

As it is usual, we indicate with $b^{-1}(k)$ and $a^{-1}(k)$ the impulse responses of these filters.

Following [15] a fast recursive scheme for obtaining the expansion coefficients in (3) is given by (the symbol * means discrete convolution):

$$c_0(k) = [b^{-1} * s](k) \tag{5}$$

$$c_{i+1}(k) = [\nu^* * c_i]_{\downarrow 2}(k) \tag{6}$$

$$d_{i+1}(k) = [\omega^* * c_i]_{\downarrow 2}(k), \tag{7}$$

where

$$\nu^*(k) = (1/2)[[a^{-1}]_{\uparrow 2} * a * u](k) \tag{8}$$

$$\omega^*(k) = (1/2)[[a^{-1}]_{\uparrow 2} * u_s * \delta_1](k), \tag{9}$$

and

$$u_s(k) = (-1)^k u(k), \ \delta_1 * a(k) = a(k-1),$$

$[a]_{\downarrow 2}(k)$ indicates $\ldots, a(-2), a(0), a(2), \ldots$ and $[a]_{\uparrow 2}(k) = a(k/2)$ if k is even and $[a]_{\uparrow 2}(k) = 0$ if k is odd.

Summing up, the coefficients of the expansion (3) are calculated with the digital filters (8) and (9).

3 The detection algorithm

With the filters previously designed we will build an algorithm for detecting epileptic events in EEG's based on the energy of the signal.

When the family $\{\psi_{k,j}(t) = \psi(2^{-j}t - k)\}$ is an *orthonormal* basis in $L^2(\mathbf{R})$, the concept of energy is linked with the usual notions derived from the Fourier theory, and the sum of the square of the coefficients of the series is the energy of the function, ie.

$$\|s\|^2 = \sum_{k,j} |d_{k,j}|^2 = \sum_{k,j} |< s, \psi_{k,j} >|^2,$$

when the wavelet decomposition is given by (4). But the wavelets we are using belong to the more general class of *biorthogonal* wavelets [15]. This means that there exists a function $\tilde{\psi}(t)$ such that

$$< \psi(2^{-i}t - k), \tilde{\psi}(2^{-j}t - l) > = \begin{cases} 2^i \text{ if } i = j \text{ and } k = l \\ \quad 0 \ \text{ otherwise.} \end{cases}$$

The family $\tilde{\psi}_{j,k}=\tilde{\psi}(2^{-j}t-k)$ is called the dual basis of $\psi_{j,k}$. Every signal $s(t)$ can be written as

$$s(t)=\sum_{j,k} d_j(k)\psi_{j,k}(t)=$$
$$=\sum_{j,k} \tilde{d}_j(k)\tilde{\psi}_{j,k}(t),$$

where

$$d_j(k)=<s,\tilde{\psi}_{j,k}>$$
$$\tilde{d}_j(k)=<s,\psi_{j,k}>.$$

In this (biorthogonal) case the energy of the signal $s(t)$ is given by

$$||s||^2=\sum_{j,k} 2^j d_j(k)\tilde{d}_j(k). \tag{10}$$

The dual $\tilde{\psi}(t)$ is itself a wavelet given by (Cf. [15])

$$\tilde{\psi}(t)=\sum_k (a*[a_s*a]_{\downarrow 2})^{-1}(k)\psi(t-k),$$

and the corresponding scaling function is

$$\tilde{\phi}(t)=\sum_k a^{-1}(k)\phi(t-k).$$

The digital filters corresponding to the dual wavelet analysis are

$$v^*(k)=(1/2)u(k) \tag{11}$$

$$\omega^*(k)=(1/2)[a_s*u_s*\delta_1](k), \tag{12}$$

and the initialization sequence is

$$\tilde{c}_0(k)=[a*b^{-1}*s](k). \tag{13}$$

The algorithms (5)-(6)-(7)-(8)-(9) and (13)-(6)-(7)-(11)-(12) allow us to calculate the coefficients involved and to get the energy function (10). Since we are using a dyadic decomposition of the range of frequencies, from a signal of M samples we have $M/2^j$ coefficients at level j. In order to get an accurate detection of the events, we uniformily distribute the "atoms" of energy in (10) –i.e. the terms $2^j d_j(k)\tilde{d}_j(k)$ – along 2^j points. Defining

$$e_j(r) = d_j(k)\tilde{d}_j(k) \tag{14}$$

for integers r in the interval $(k-1)2^j < r \le k2^j$, the energy in each resolution level $j = 1,\ldots,N$, is

$$E_j = \sum_{r=1}^{M} e_j(r),$$

and the energy in each sampled time $r = 1,\ldots,M$ is

$$E(r) = \sum_j e_j(r).$$

Different types of epileptic events can be characterized for the values $e_j(r)$ in different resolution levels. The detection is made when the value $e_j(r)$ is greater than a threshold D_j defined for each level.

4 Results and Discussion

EEG records from depth electrodes (diameter 1.8 mm) were analysed. The figures 1-5 show the results obtained by processing EEG signals sampled at 256 Hz, from different patients which are candidates to quirurgical treatment. As it was mentioned, different epileptic events appear in different resolution levels. For example, the spikes appear in the level $j = 3$, and the waves corresponding to the spike-and-waves in the level $j = 5$. In the Figs. 1 and 2 the detection of a spike-and-wave and a spike respectively, are shown. Fig. 3 ilustrates the results obtained with the proposed method in the detection of a train of spike-and-waves and an isolated spike. For comparison, Fig. 4 shows the results obtained with other train of spike-and-waves and base activity. In all the previous cases, the results were obtained with the wavelet-based detector plus suited thresholds, as it was explained. The value of the thresholds are choosen according to the statistical analysis of the base activity. In each case a constant value is assigned when the energy function is greater than the corresponding threshold. More details of the EEG signal obtained by assigning a proportional value to the energy function when this function overcome the threshold, as Fig. 5 shows.

One of the typical characteristics of the wavelet analysis is the freedom in the choice of the basic wavelet. As it was pointed out, an important part of the answer to the question about the wavelet choosen are the results obtained with it. The algorithm proposed in this paper is capable to detect spikes, spikes-and-waves, base activity, and slow waves. The basis of such capacity are in the different levels of analysis in the frequency domain. In fact, in the scheme proposed we analyse eight frequency bands (octaves) covering from high to low frequencies. The characteristic events presented in the EEG can be searched in the adequate level. This fact differences the proposed detector from the previous ones enumerated in the Introduction. A complete picture of an EEG processed with our algorithm is shown in Fig. 6. We can observe the 8 intervals of frequency and the reconstructions of the signal in each one. In order to show that the information involved in the signal is kept in the wavelet analysis, the reconstruction of the signal using the 8 levels is shown in the last row of Fig. 6. We have shown in this paper that our treatment of the information contained in the proposed wavelet analysis lead to an efficient algorithm for detecting epileptic events.

5 Conclusions

The results presented in this paper show the capabilities of multiresolution wavelet analysis for the detection of nonstationary phenomena in EEG signals from depth electrodes. We will enumerate the main facts of our analysis.

1. Different types of epileptogenic events have different frequency localization, and correspond to different levels in the multiresolution.

2. As Figure 6 clearly shows, the complete information contained in the EEG signal can be found in the eight multiresolution levels. In fact, from these levels it is obtained the perfect reconstruction of the EEG signal shown in the last row of the figure. This fact demonstrates that no information is lost, and then, with an adequate use of this information splitted in the eigth levels the detection of transients is possible.

3. The energy treatment explained in Section 3 allows an efficient detection of characteristic epileptic events. This fact is independent of the basic wavelet, and remain valid for any dyadic multiresolution.

4. The algorithm presented in the paper shows that the computational efficiency of the multiresolution can be used in event-detection problems, despite the dyadic decomposition of the frequency axes.

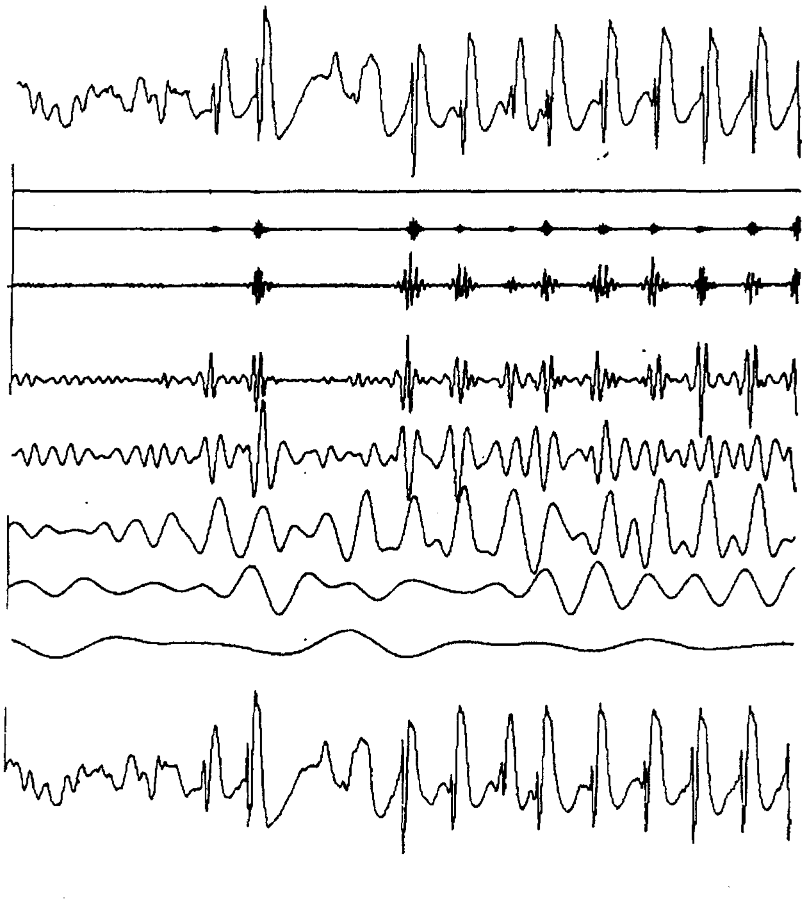

Figura 6

The previous remarks suggest that computational techniques based on wavelet theory may be incorporated in the automatic analysis of EEG signals from depth electrodes in order to deal with the problem of extraction features containing relevant information.

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