Pro Mathematica Vol. XII, Nos. 23-24, 1998

USING WAVELETS IN APPLIED ELECTROMAGNETICS: AN ALGORITHM FOR EFFICIENT PATTERN SYNTHESIS OF LINEAR APERTURE ANTENNAS .

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Abstract

Pattern synthesis of linear aperture antennas is a classical problem in Applied Electromagnetics with a great number of resolving methods. In this paper, a new algorithm that uses wavelets to compute the solution is presented. The new method reaches a good approximation of the aperture field from the antenna radiation pattern, using the Haar wavelet basis. Results prove the performance of the proposed method.

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1. Introduction

During the last decade, more and more mathematical methods using wavelets have been developed. Information Technologies employ them in Signal Processing, Speech Characterization, Filtering and a great number of different applications. Electromagnetic Science is actually in the wavelet road:. methods to compute the solution to equations had been created all around the world. In this paper, a new wavelet application is presented: an algorithm for pattern synthesis of linear aperture antennas. Results of different cases are presented, showing the evolution of the constructive algorithm.

2. Pattern Synthesis

The IEEE Standard Definitions of Terms for Antennas defines the antenna or aerial as "a means for radiating or receiving radio waves" [1]. So that, the antenna is the transitional or adaptation structure between free-space and a guiding device. This guiding device may be a coaxial line or a waveguide (hollow pipe) and it is used to transport electromagnetic energy from the transmitting source to the antenna (transmitting antenna), or from the antenna to the receiver (receiving antenna).

In addition to that function, it is usually to require an antenna to optimize or accentuate the radiation energy in some directions and suppress it in others. Thus the antenna must act as a directional device in addition to a probing device.

These directional performances are showing by the radiation pattern, a graphic representation of radiated power versus radiation angle. In other words, this kind of diagrams shows the directions where the antenna radiates (or receives, the antenna is a reciprocal device) the more electromagnetic field and the directions where the radiated field is null or little.

Antennas can be classified into two great groups: wire antennas, and aperture antennas. The difference between the two possibilities is determined by their geometry and analysis methods.

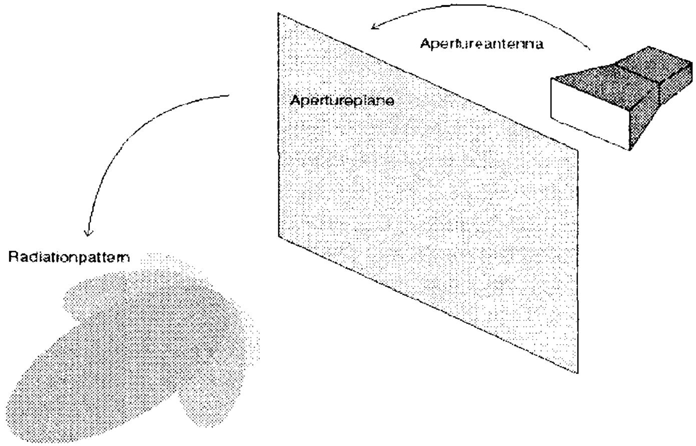

Figure 1: Schematic procedure to analyze aperture antennas

The analysis of aperture antennas can be made using a mathematical assumption: there is a plane near the radiation source, and perpendicular to the travelling wave propagation, called 'aperture plane' (figure 1). It is demonstrated the relationship between electromagnetic fields in that 'aperture plane' and radiation pattern. So that, the knowledge of distribution of fields in the aperture is enough to analyze an aperture antenna (e.g. to determine its radiation pattern), and there are various procedures to calculate radiation patterns from aperture fields [2]. In fact, radiation pattern can be obtained from aperture field in the following sense, written for the case of rectangular aperture:

$$L(\theta,\ \phi) = cos\,\theta \cdot cos\,\phi \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \left(-2\hat{n}\,x\,\vec{E}_{ap} \right) e^{\,j\beta(x' sin\,\theta\,cos\,\phi + y' sin\,\theta\,sin\,\phi)} dx'dy'$$

where:

$\vec{E}_{ap}$ is the aperture field.

$\hat{n}$ is the vector orthogonal to the aperture plane.

θ, ϕ are the angular coordinates of radiation direction.

x', y' are Cartesian coordinates in the aperture plane.

a, b are the linear dimensions of the aperture plane.

This paper is about the inverse problem. We know a desired radiation pattern related to an aperture antenna. But we do not know how the antenna is: the final objective is to design the radiating system. In order to develop an antenna design, we must compute the field distribution in the aperture plane which radiated field best adjust the radiation pattern in least square sense. This is called pattern synthesis of aperture antennas. And there is no immediate solution to this problem, although there are a lot of different methods to reach it.

A new constructive wavelet algorithm that uses wavelets in pattern synthesis of aperture antennas is presented. It solves that classical problem using wavelets, and there is its newness.

3. Wavelets in Electromagnetics

The modern wavelet theory was born during the last quarter of XX century, searching for new studies on Image Processing. It could be said that Yves Meyer and Stephan Mallat were the fathers. Some decades before (1910), Alfred Haar published a paper presenting the first wavelet basis, although he did not know how a wavelet is. After that, Shannon enunciated his sampling theorem, where a wavelet appears. But he had not notice of that. Before, Yves Meyer found the first modern wavelet in 1984, trying to prove that there were no more wavelets but the Haar one.

During 80's and 90's a lot of works on wavelets was presented due to mathematicians, engineers and physicians, and at the some time an increased number of practical applications have been appeared. Nowadays, it can be said that researchers apply wavelets in the resolution of problems that was traditionally solved by Fourier Transform, taking advantage in the performance of wavelets to represent non-stationary signals [3].

Looking to Applied Electromagnetics, there are a lot of scientific works based on wavelet techniques, the most of them for solving the integral equation of the Moment Method [4].

The knowledge of current density distribution is necessary to solve the whole antennas or scattering problems. The way to reach a solution is to enunciate an integral equation where that current distribution is the unknown, and solve it. From the current it is possible to calculate the electromagnetic fields. The integral equation can be known as EFIE, when it is based on electric field, or MFIE, when it is based on magnetic field, and both uses the boundary conditions.

Wavelets have been used to solve the Method of Moments integral equations in frequency domain. The unknown variable is represented as wavelet function series and the difference between both sides of the equation is forced to be orthogonal to a set of wavelet functions (called test functions). In this way, a wavelet basis matrix representation of the operator is reached. In most of cases that matrix is localized and can be reduced by a threshold procedure. The results by methods like those are good.

Baharav and Leviatan have presented some ways for the Method of Moments solution that uses wavelets, as IMC (Impedance Matrix Compression) [5] or iterative IMC [6], and other techniques like SMT (Source Model Technique): scattering analysis using fictitious wavelet array sources [7].

A lot of scientific works have been published showing the performance of wavelet procedures to solve Electromagnetic integral equations [8] [9] [10], or using wavelet in antenna analysis [11].

4. The Constructive Wavelet Algorithm

The constructive wavelet algorithm can be divided into two stages (figure 2) for its explanation: the first stage will be called start stage, and the second one, construction multistage. The start stage initializes the algorithm and fixes its objective. The construction multistage can be repeated to reach a solution containing the desired detail level. Here, we present the algorithm for two-dimension aperture synthesis.

A. Start Stage

This stage is divided into two steps.

a. A wavelet basis on the whole aperture is constructed. This basis will represent the electromagnetic field. At this point, we select which wavelet family will be used. Shown results have been computed using Haar wavelets, because its simplicity to programming. Haar function was the first wavelet in Mathematics History [12].

A wavelet family is defined by its scale function, ϕ, and its wavelet functions (sometimes called wavelet "mother"), ψ. For computing the whole aperture, we define both ϕ and ψ for covering all the aperture space.

b. A set of functions is selected from the wavelet basis for initializing the algorithm. This set will be called $k = 0$ basis.

B. Construction Multistage

This stage is divided into five steps.

c. Each wavelet from k basis, located at the aperture plane, will create a radiated field. The relationship between aperture field and radiated field is linear; so that if an aperture field $\vec{E}_{ap}$ produces a radiated field $\vec{E}_r$, then a field $c\vec{E}_{ap}$ will produce a radiated $c\vec{E}_r$

The computation of radiated field is made from the aperture field by the following equation:

$$\vec{E}_r = \cos\theta \cdot \int_{-\frac{a}{2}}^{\frac{a}{2}} (-2\hat{n} \; x \; \vec{E}_{ap}) \cdot e^{j\beta x' \sin\theta} dx'$$

where:

E_r is the radiated field.

E_{ap} is the aperture field.

n is the unitary vector orthogonal to the aperture plane.

a is the length of the aperture.

β is the wave number.

θ is the observation angle from the axial direction.

Known the radiated field due to each function from k basis and the prescribed radiation pattern, functions from k basis are pounded to minimize the error in the sense of least square. We called c_k the coefficient series related to functions in k basis.

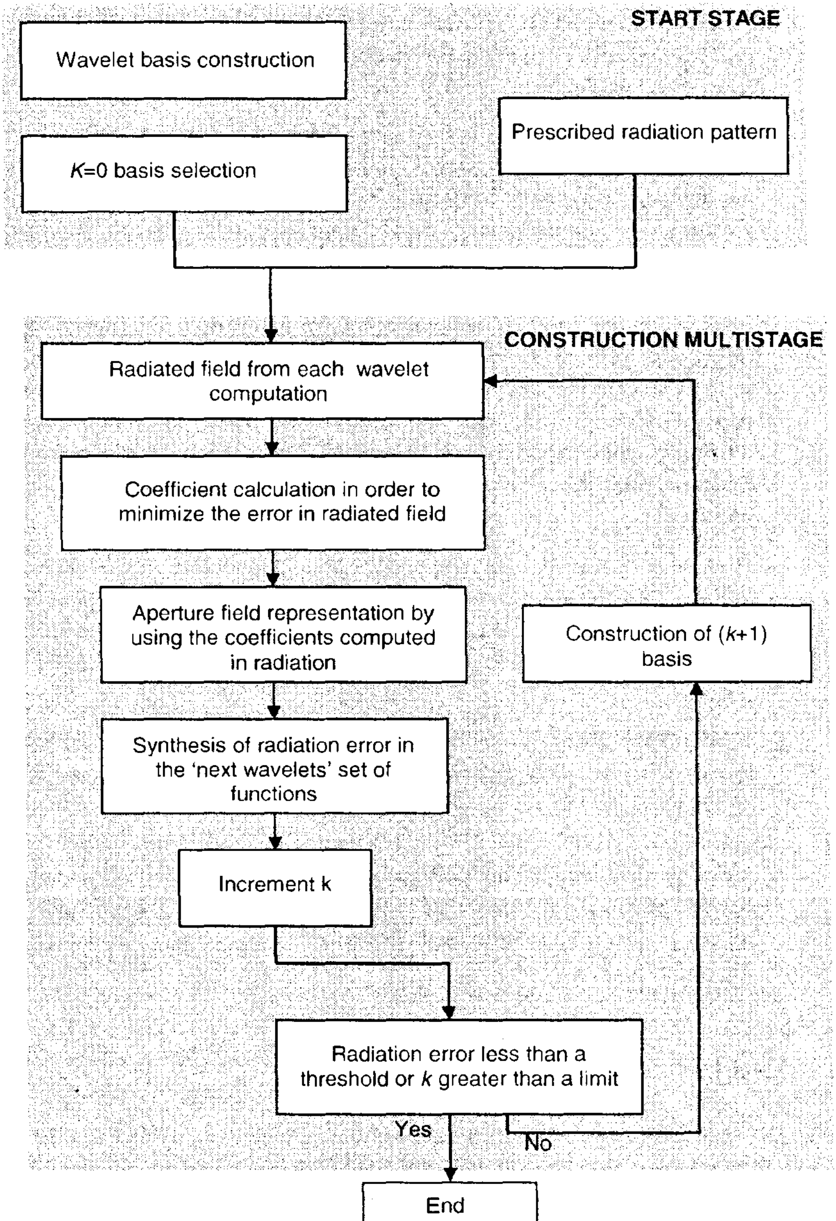

Figure 2: The Constructive Wavelet Algorithm

d. Stage k aperture field is then computing using wavelets on k basis and c_k coefficients. This aperture field is expressed as the pounded sum of the functions.

e. Then, we must to decide if the computed results are good enough or if it is necessary another stage. In this way, radiation error vector is synthesized using the "*next wavelets*". The "*next wavelets*" are the functions in the following detail level of each selected wavelet function in k basis.
   In other words, the way of construction of wavelet basis, scaling and translating the wavelet function, forces the representation of the target function to be made in different detail level. Each scale defines a detail level, and all the translations of the same scaled-function belong to the same detail level. So those, to reach a more accurate representation of the target function, the wavelets proportioning more details may be used. And they are the "*next wavelet*".

f. If error is less than a threshold or if k is greater than the limit step number, the algorithm will stop. The selected wavelet family and the aperture dimensions define the limit of k: for k too great, resolution of scaled functions will be too small and will lead to numerical errors.

g. If algorithm must to go on, the most important wavelets in the radiation error synthesis will be added to the k basis to create the $(k + 1)$ basis. Then, another construction stage will begin, from point *c*.
   The end of the algorithm, at point *f*, gives us a set of wavelets and a set of coefficients. They can construct an efficient synthesis of aperture field, obtained from a known radiation pattern. And this is the initial problem.

5. Results

The first result we present is a validation of the new algorithm. It is best known that a Taylor distribution aperture field radiates a low-level side lobe radiation pattern. Figures 3 and 4 shows the aperture field and its corresponding radiation pattern. The radiation pattern that appears in figure 4 will be the prescribed one for the first example.

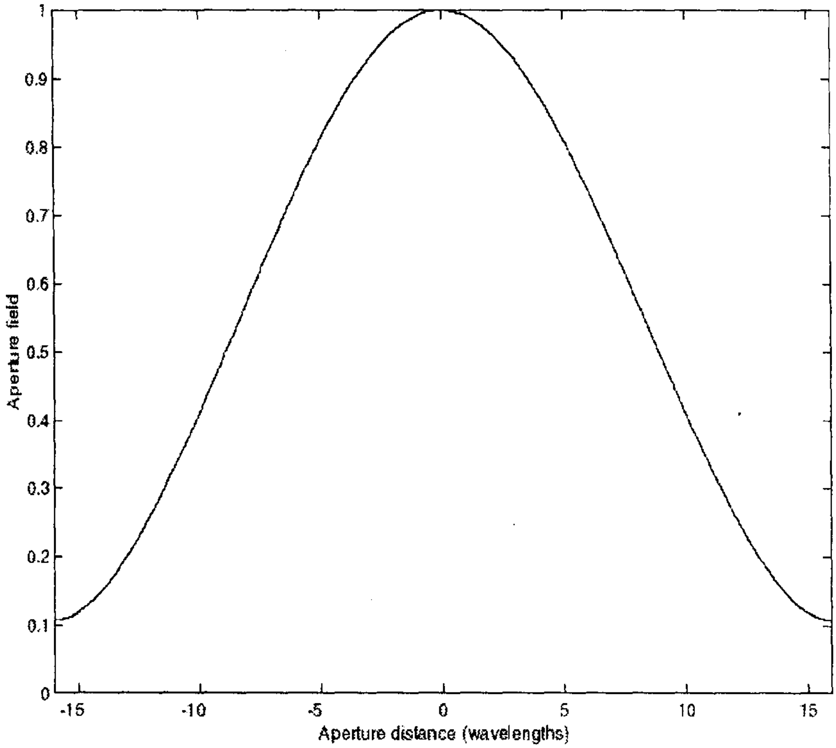

Figure 3: Aperture field that creates a Taylor radiation pattern.

So that, beginning from a Taylor radiation pattern as a prescribed pattern, the algorithm would approximate a cosine field aperture if it works in a right way. After the first stage, the situation is the one shown in the figures 5 and 6. Dashed line in both figures is the target, and continuous line represents the wavelet algorithm results.

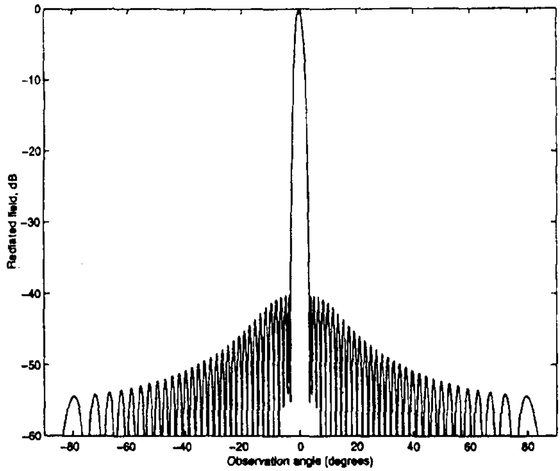

Figure 4: Taylor radiation pattern.

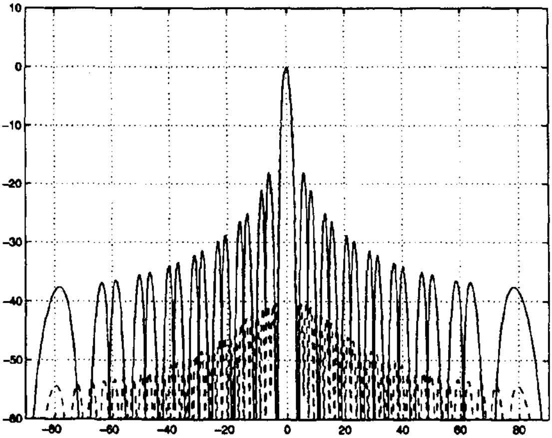

Figure 5: Radiation pattern after stage 1.

Figure 5 shows a good agreement in the main lobe, but side lobes too high. The reason of that can be understood looking to figure 6.

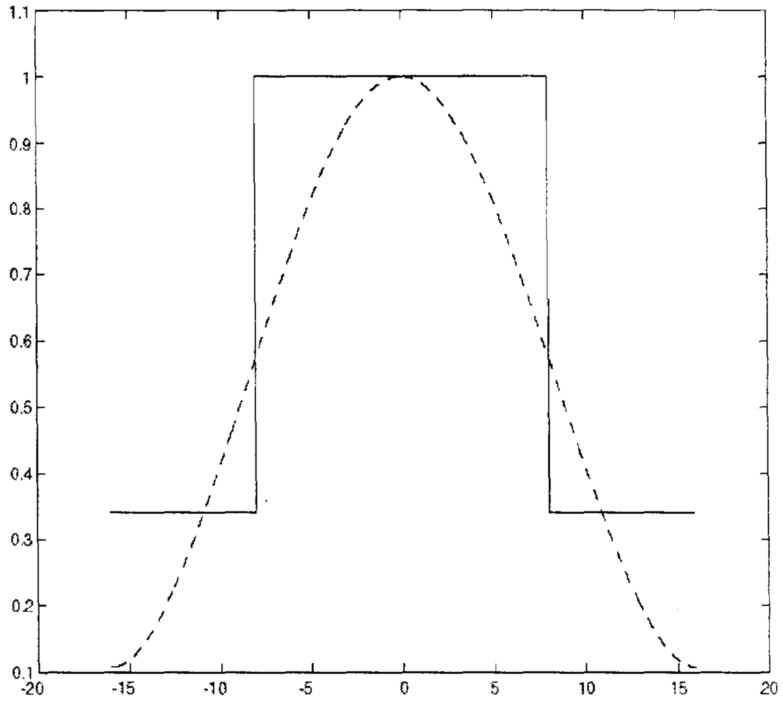

Figure 6: Aperture field diagram after stage 1.

After three stages, the amplitude of side lobes is going to fix the prescribed radiation pattern as it can be seen at figure 7. Figure 8 shows the related aperture field.

After five stages, the agreement between the prescribed and the computed radiation pattern is better, and the same thing can be said for aperture field. Figures 9 and 10 shows this situation.

After seven stages, the algorithm is stopped, and the last situation is shown in the figures 11 and 12. Evidently, the radiation pattern errors are minimums and the synthesized aperture field fits the desired one.

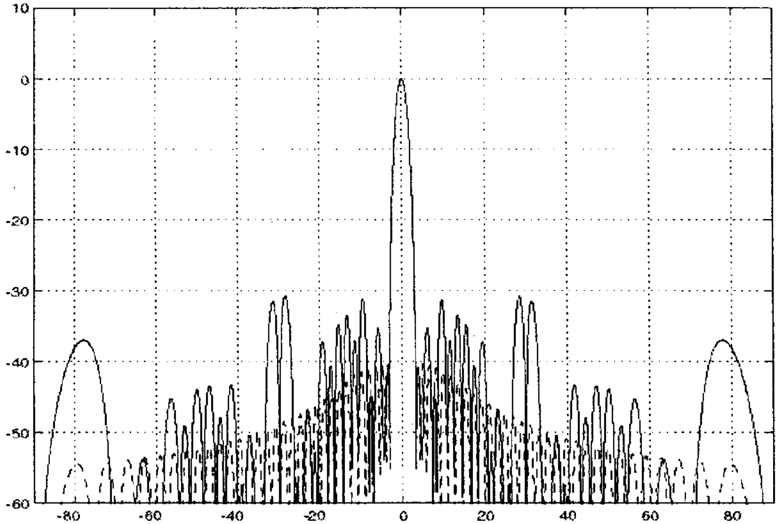

Figure 7: Radiation pattern after stage 3.

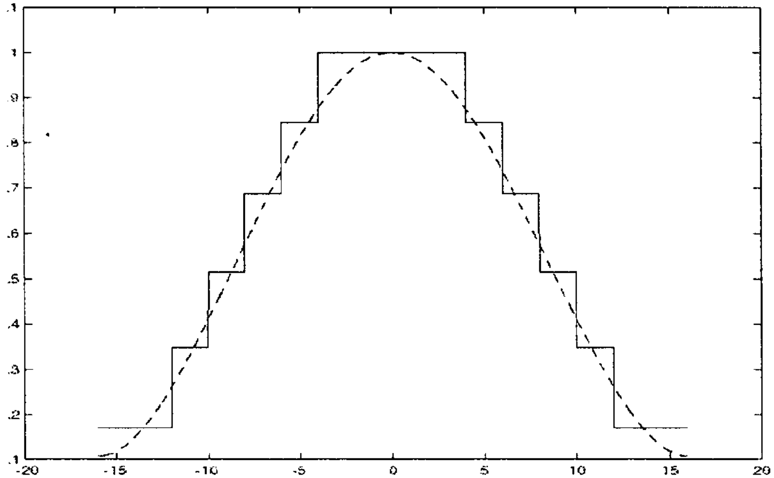

Figure 8: Aperture field diagram after stage 3.

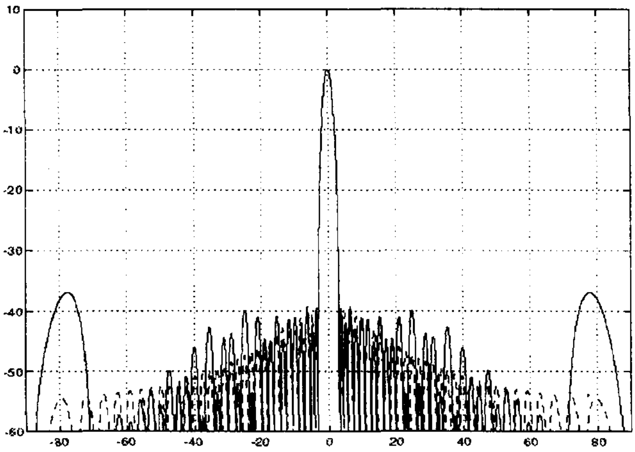

Figure 9: Radiation pattern after stage 5.

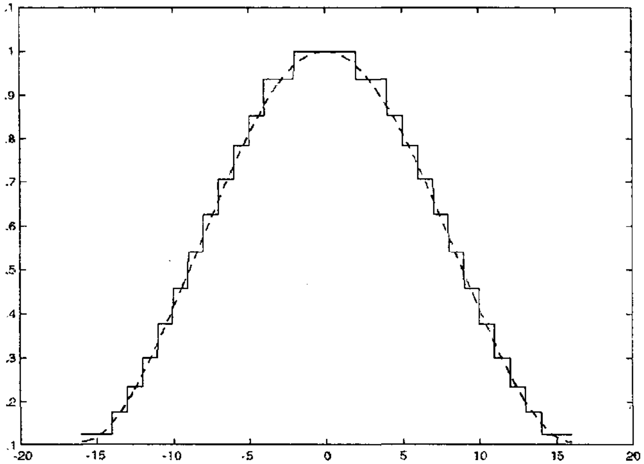

Figure 10: Aperture field diagram after stage 5.

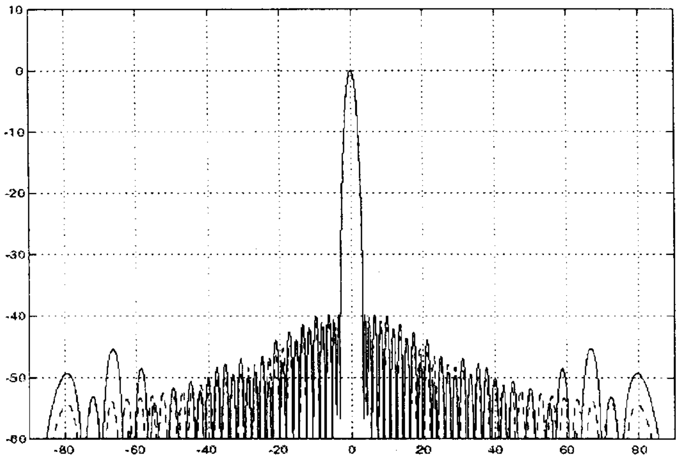

Figure 11: Radiation pattern after stage 7.

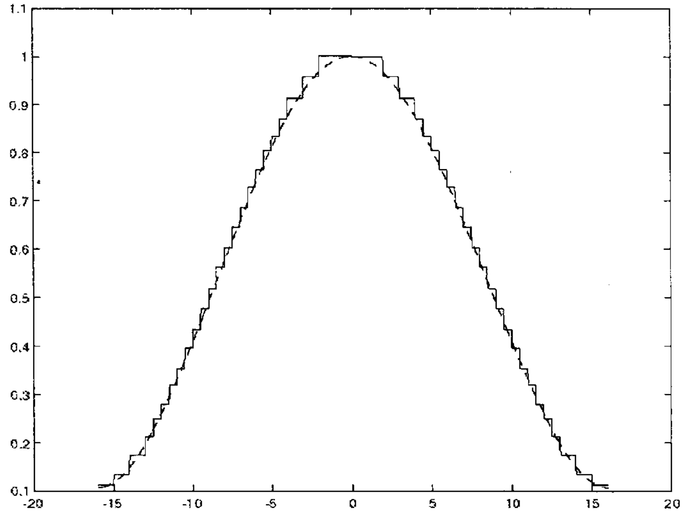

Figure 12: Aperture field diagram after stage 7.

After seven stages, the algorithm reaches a good approximation of the target aperture field. Looking to the result, it can be said that the algorithm really obtains the synthesis of the aperture field.

Another proof of the performance of the method may be a Taylor diagram with a 10 degrees disappointed. In this example, the capability of phase adjustments of the algorithm will confirm the predicted success. Phase movements in the aperture make the disappointment of radiation beam while field amplitude becomes static. To reach the phase adjusts the algorithm needs more stages as it can be seen looking to the following figures. Figures 13 and 14 shows the initial stage.

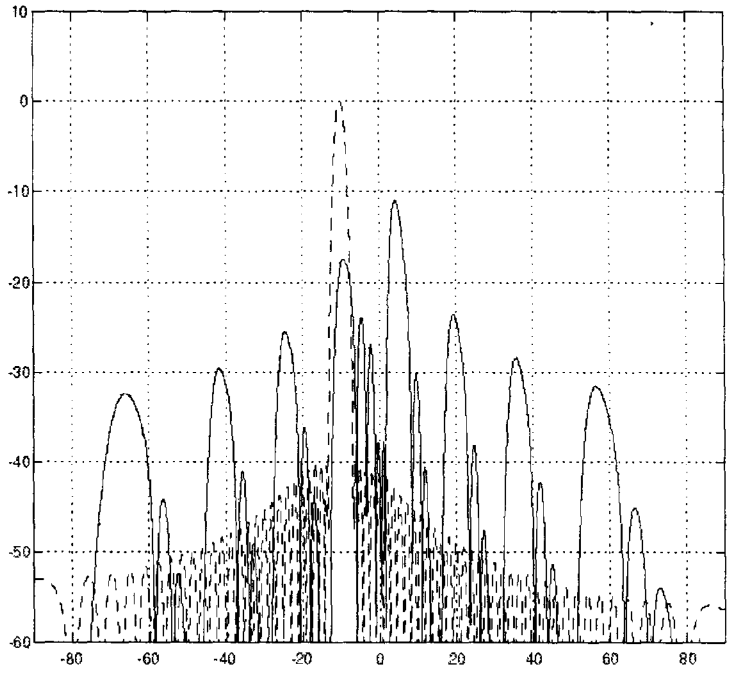

Figure 13 Radiation pattern after 1 stage.

The main lobe is adjusted after 4 stages, as figure 15 shows, but this radiation pattern has great side lobes. Figure 16 contains the amplitude and phase of the aperture field after 4 stages.

Figure 17 and 18 shows the way the algorithm tries to reduce the magnitude of side lobes. This is the situation after 7 stages.

After 9 stages, the greater side lobe has been reduced to be 27 dB below the main lobe. Figures 19 and 20 shows this stage.

After 11 stages, the solution obtained by the wavelet algorithm is shown in figures 21 and 22.

At least, figures 23 and 24 shows the results at the point the algorithm is stopped. Radiation pattern has a very good agreement with the disappointed Taylor pattern, as it is shown in figure 23. The synthesized aperture field is shown in figure 24.

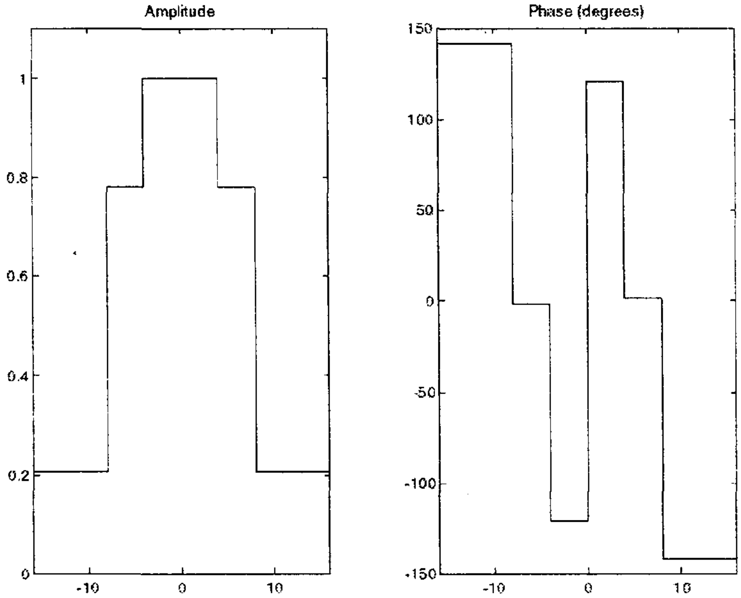

Figure 14: Aperture field after 1 stage.

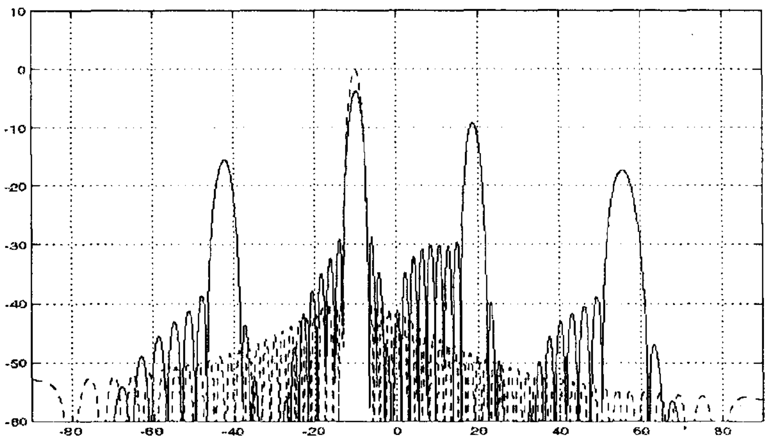

Figure 15: Radiation pattern after 4 stages.

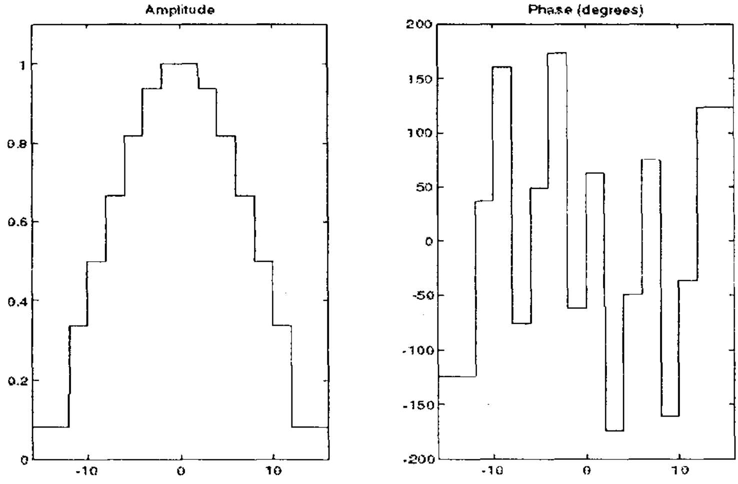

Figure 16: Aperture field after 4 stages.

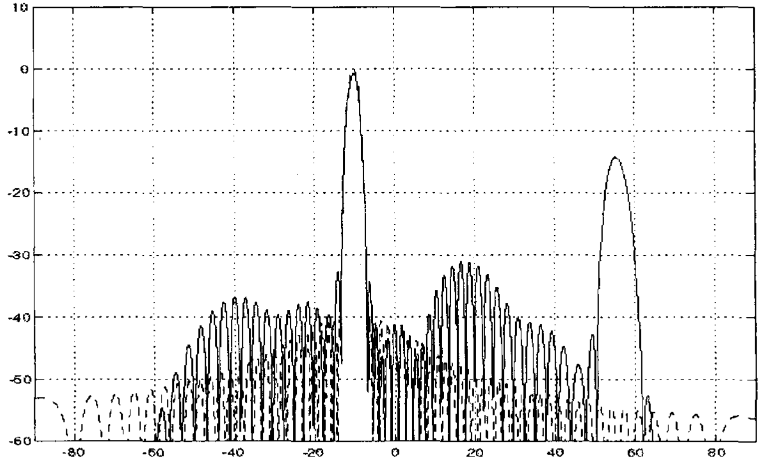

Figure 17: Radiation pattern after 7 stages.

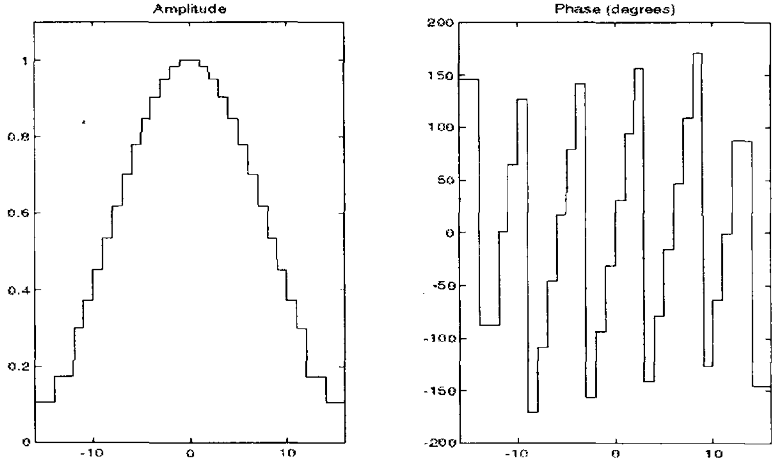

Figure 18: Aperture field after 7 stages.

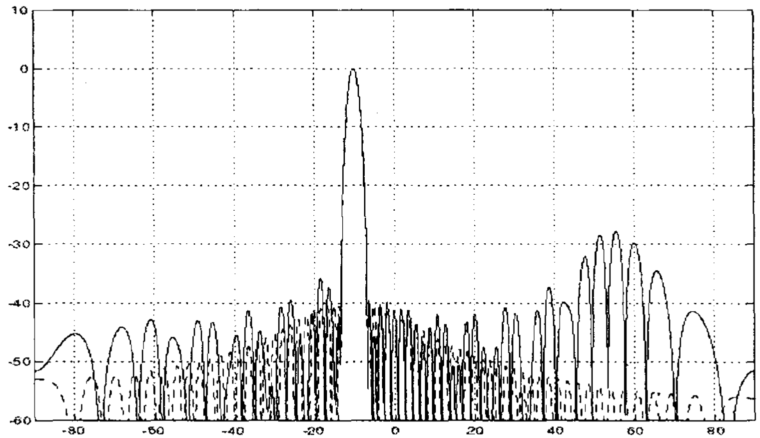

Figure 19: Radiation pattern after 9 stages.

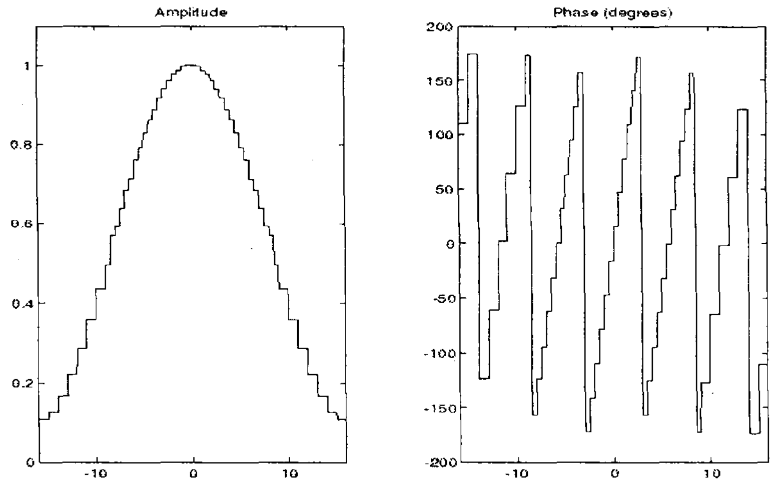

Figure 20: Aperture field after 9 stages.

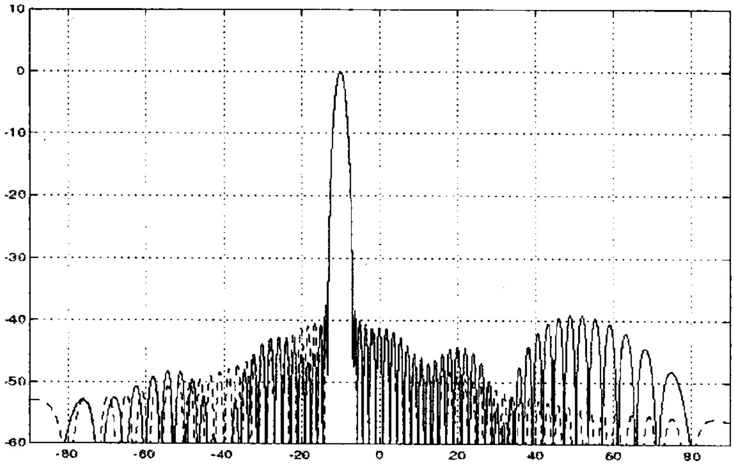

Figure 21: Radiation pattern after 11 stages.

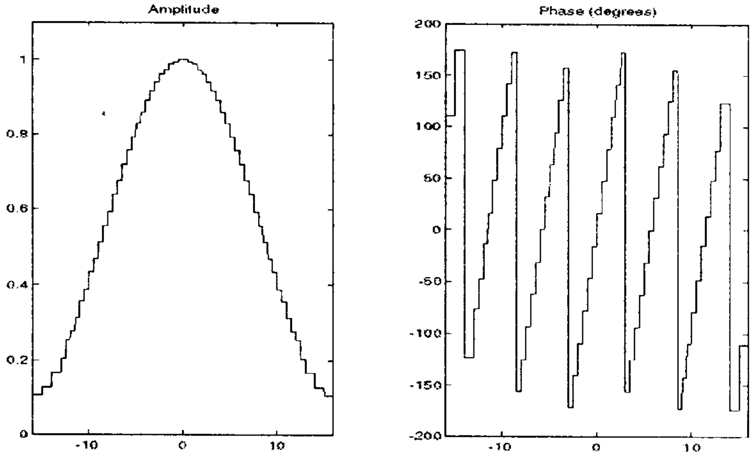

Figure 22: Aperture field after 11 stages.

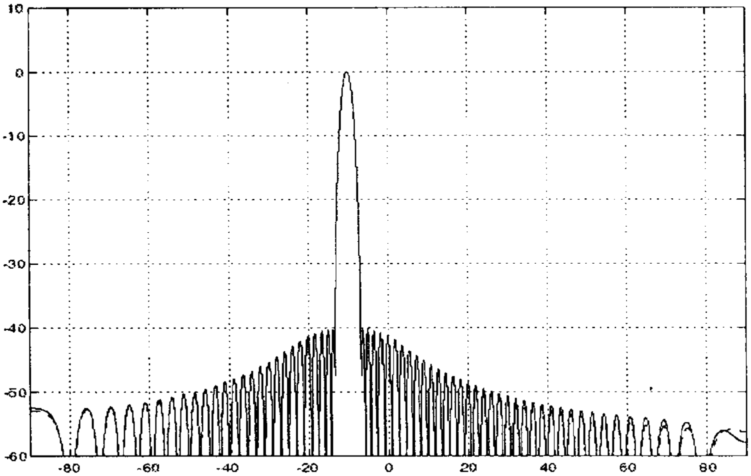

Figure 23: Radiation pattern after 12 stages.

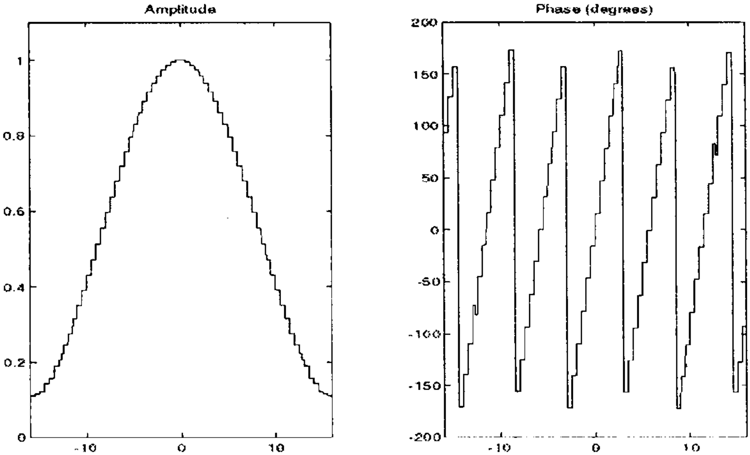

Figure 24: Aperture field after 12 stages.

6. Conclusions

A new algorithm that uses wavelets for the synthesis of aperture antennas has been presented. Constructively, the algorithm adds wavelet functions to a basis in order to approximate the target radiation pattern by the radiation due to that basis. With those functions the field in the aperture can be constructed.

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